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by

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ABSTRACT

A numerical method for two-dimensional cavitating flow is developed for the flat plate. The linearized boundary value problem is restated as a set of coupled integral equations. The integral equations are approximated numerically. The numerical approximation is executed by a Fortran IV computer program. The computed results are compared to the analytic solution. This method should provide insight into developing a method for three-dimensional cavitating flows and is readily extendable to cambered profiles.

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NOMENCLATURE

A = Cavity Area

c = chord length

$[C]$ = coefficient matrix

c_{ji} = element of coefficient matrix

C_p = pressure coefficient

C_L = lift coefficient

f = location of vortex control points

f_s = location of source control points

h = locus of points describing mean camber line and cavity surface

ℓ = cavity length

M = number of vortex elements

N = number of source elements

P = pressure

P_∞ = pressure at infinity

q = velocity

$q(\xi)$ = source distribution

u,v = perturbation velocity components

U_∞ = inflow velocity

w = complex velocity = $u-iv$

x,y = coordinate system

$x_{\ell i}$ = lower boundary of vortex or source element

z = complex plane = $x + iy$

$(\bar{\ })$ = quantity with a bar is a dimensional quantity

$(\ )$ = no bar indicates a nondimensional variable

$(\ )^+$ = variable to be evaluated at $x = 0^+$

$(\ )^-$ = variable to be evaluated at $x = 0^-$

$(\ )_x$ = variable to be differentiated wrt x

α = angle of attack

$\gamma(\xi)$ = vortex distribution

ζ = mapped plane $= \xi + i\eta$

ξ, η = dummy coordinate system for integration and see ζ above

ρ = fluid density

σ = cavitation number

ϕ = velocity potential

φ = perturbation velocity potential

n = degree of singularity

CHAPTER I

INTRODUCTION

The objective of this investigation is to develop a numerical solution to cavitating flows which can be compared to a known analytic solution. Therefore, the cavitating two-dimensional flat plate in steady flow has been chosen for this investigation. Geurst [1,2,3] has given solutions for the linearized problem for both partial and super cavitation.

The purpose in considering this problem is to determine and overcome the numerical difficulties associated with this problem. Adequately solving the two-dimensional problem is a preliminary step in developing a numerical solution for the far more difficult three-dimensional and unsteady flow problems.

Further the numerical method can be applied to foils of different camber lines. Geurst [2] has obtained a camber line solution for the case of parabolic camber in partial cavitation only.

Widnall [4] developed a numerical method for three-dimensional, unsteady, supercavitating flows. However, Widnall used assumed cavity lengths and demonstrated that, for long cavities, lift coefficients were relatively insensitive to cavity length. Clearly for short super cavities, on the order of chord length, and partial cavities, a proper statement of cavity termination is required for the proper solution of cavitating flows.

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In this respect this report is a preliminary investigation into the inclusion of cavity termination in cavitating flows. The results of this report should be applicable to three-dimensional and unsteady cavitating flows.

The solution of two-dimensional, cavitating flows presented herein envolves five distinct steps. These are:

1) statements of the linearized boundary value problem,

2) solution of the boundary value problem in the form of coupled integral equations in source and vortex distributions,

3) a numerical approximation to the integral equations,

4) execution of the numerical approximation via FORTRAN computer program,

and

5) validation of the numerical results.

Linearizing the boundary value problem and rewriting the problem in the form of integral equations is not the only method of solution. Another possible approach is the finite difference method. Jeppson [5,6] and Mogel and Street [7] have taken the finite difference method as far as a circular disk perpendicular to the main stream with an infinite cavity. The major disadvantage of the finite difference method is the large programing and computational effort required.

The integral equation method for the linearized problem represents a far smaller computational effort. It is for this reason, plus the existence of the analytic solution, that the integral equation method is used. For an excellent discussion of the various methods of solution see Birkhoff [8] .

The following chapters of this report will discuss the stated steps of the solution.

CHAPTER II

THE LINEARIZED BOUNDARY VALUE PROBLEM

Geurst [1,2,3] has presented a development of the linearized boundary value problem for both partial and super cavitation. A similar development is given here, except that the coordinate system and the nondimensionalization of variables differ. These differences are strictly a matter of convenience. Geurst's method of solution relies on a series expansion of the complex velocity in the neighborhood of infinity. This in turn requires that the boundary value relations to local slope on the foil and cavitation number be stated at infinity.

The neighborhood of infinity is not convenient to computer solutions. Therefore the boundary value relations to local slope on the foil and cavitation number need to be stated on the foil and cavity.

The coordinate system and the foil-cavity relation to the coordinate system are shown in figure 1(a). The inflow velocity, $\bar{U}_\infty$, is assumed to be parallel to the x axis and uniform in the y and z directions. The angle of attack, α , is taken to be the nose-tail line for cambered profiles.

First the steady flow form of Bernoulli's equation is written between infinity and another point in the flow:

$$(1) \qquad \frac{\bar{P}_\infty}{\bar{\rho}} + \frac{1}{2}\bar{U}_\infty^2 = \frac{\bar{P}}{\bar{\rho}} + \frac{1}{2}\bar{q}^2, \qquad \bar{q} = |\vec{\nabla}\bar{\phi}|$$

This equation is now rewritten in the form of a nondimensional pressure coefficient.

$$(2)\qquad C_p = \frac{\bar{P}_\infty - \bar{P}}{\frac{1}{2}\bar{\rho}\bar{U}_\infty^2} = \left(\bar{q}^2/\bar{U}_\infty^2 - 1\right).$$

On the cavity the pressure coefficient is the cavitation number, σ. The velocity potential can be written as the sum of the potential due to the inflow velocity plus a perturbation potential:

$$(3)\qquad \bar{\phi} = \bar{U}_\infty \bar{x} + \bar{\varphi}$$

Where

$\bar{\varphi}$ = perturbation velocity potential.

From equation (3) the $\bar{x}$ and $\bar{y}$ components of velocity are:

$$(4)\qquad \frac{\partial\bar{\phi}}{\partial\bar{x}} = \bar{U}_\infty + \frac{\partial\bar{\varphi}}{\partial\bar{x}} = \bar{U}_\infty + \bar{u}$$

$$(5)\qquad \frac{\partial\bar{\phi}}{\partial\bar{y}} = \frac{\partial\bar{\varphi}}{\partial\bar{y}} = \bar{v}$$

Where

$\bar{u} = \bar{x}$ component of the perturbation velocity

$\bar{v} = \bar{y}$ component of the perturbation velocity.

It is assumed that the perturbation velocity is very much less than the inflow velocity, that is:

$$\bar{u}, \bar{v} << \bar{U}_\infty$$

On the cavity we now have:

$$\bar{q}_c^{\,2} = [\bar{U}_\infty^2 + 2\bar{u}_c\bar{U}_\infty + \bar{u}_c^2 + \bar{v}_c^{\,2}]$$

using(6)

$$(9) \qquad \frac{\bar{q}_c^{\,2}}{\bar{U}_\infty^2} = 1 + 2\bar{u}_c/\bar{U}_\infty$$

Thus equation (2) for the cavity becomes:

$$(8) \qquad u_c = \sigma/2$$

Where

$$u_c = \bar{u}_c/\bar{U}_\infty$$

and $\bar{U}_\infty$ is now used as a nondimensionalization constant.

Equation (8) is the statement of the linearized boundary condition on the cavity surface. The next step is to obtain the boundary condition on the wetted surface of the foil.

On the foil wetted surface the boundary condition is flow tangency.

$$(9) \qquad \frac{\bar{v}}{\bar{U}_\infty + \bar{u}} = \frac{d\bar{h}}{d\bar{x}}$$

Which becomes

$$\frac{\bar{v}}{\bar{U}_\infty} + \frac{\bar{u}\bar{v}}{\bar{U}_\infty} + \text{higher order terms} = \frac{d\bar{h}}{d\bar{x}}$$

or

$$(10) \qquad v = \frac{\bar{v}}{\bar{U}_\infty} \simeq \frac{dh}{dx}$$

Also, it is assumed that the foil is very thin compared to the chord length such that the boundary condition in equation (10) can be applied to the mean camber line.

Further the entire foil cavity system is collapsed to the $\bar{x}-\bar{z}$ plane and the boundary conditions applied on the $\bar{x}-\bar{z}$ plane with all length dimensions nondimensionalized by the chord length, figure 1(b). Another condition to be satisfied is cavity termination.

Geurst [2] proved that for the linearized problem, the re-entrant jet and Riabouchinsky models for cavity termination reduce to a statement that the cavity closes at its end. This condition can be written as

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(11) $$\int_0^{\ell}\left[\left(\frac{dh}{dx}\right)_{cavity}-\left(\frac{dh}{dx}\right)_{foil}\right]=0$$

or

(12) $$\int_0^{\ell}\left[h_x^+ - h_x^-\right]=0$$

The next condition to be satisfied is the condition at the trailing edge of the foil (x = 1). This condition requires that there be no pressure jump accross the foil at the trailing edge.

(13) $$C_p^+(1) - C_p^-(1) = 0$$

In summary the conditions that the solution must satisfy are:

(C1) $u^+ = \sigma/2$ $\qquad 0 \leq x \leq \ell$

(C2) $v = h_x$ $\qquad$ on wetted surface of the foil

(C3) $$\int_0^{\ell}\left[h_x^+ - h_x^-\right]=0$$

(C4) $$C_p^+(1) - C_p^-(1) = 0$$

The last condition is that the perturbation potential satisfies Laplace's equation:

(C5) $\nabla^2 \phi = 0$.

The next chapter will give a solution to Laplace's equation which satisfies the above boundary conditions.

CHAPTER III

THE INTEGRAL EQUATION FORMULATION

This chapter presents the development of the integral equation formulation of the problem. This development proceeds from the solution for the velocity field induced by a two-dimensional distribution of vorticies and sources in space. Then it can be shown that this velocity field will satisfy the boundary conditions. This results in a set of coupled integral equations.

First consider the flow to be in the complex z plane,

Where:

$$z = x + iy$$

and the complex velocity is given by:

$$w(z) = u - iv$$

Then the appropriate general form of the solution is [9]:

$$(14) \qquad w(z) = \frac{1}{2\pi}\int_0^{l} \frac{q(\xi)}{z-\xi}\,d\xi - \frac{i}{2\pi}\int_0^{1} \frac{\gamma(\xi)}{z-\xi}\,d\xi$$

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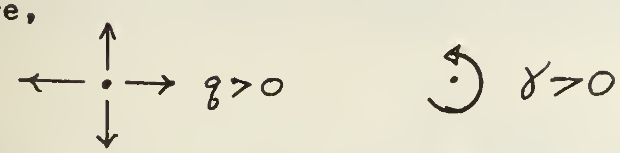
or

$$(15) \qquad W(z) = \frac{1}{2\pi i}\int_0^a \frac{(-\gamma - iq)}{\xi - z}\, d\xi$$

where,



$q(\xi)$ = source distribution over the cavity

$\gamma(\xi)$ = vortex distribution over the foil

$$a = \begin{cases} 1 & \text{for a partial cavity} \\ \ell & \text{for a super cavity} \end{cases}$$

$q(\xi) = 0$ for $\ell < \xi$ for a partial cavity

and

$\gamma(\xi) = 0$ for $1 < \xi$ for a super cavity.

Plemelj's formulas can be applied to equation (15) as follows [10].

$$(16) \qquad W^{+}(x) - W^{-}(x) = -\gamma(x) - iq(x)$$

$$(17) \qquad W^{+}(x) + W^{-}(x) = \frac{1}{\pi i}\,⨍_0^a \frac{-\gamma(\xi) - iq(\xi)}{\xi - x}\, d\xi$$

When equations (16) and (17) are added together and the real part is taken, the result is:

$$u^{+}(x) = -\frac{\gamma(x)}{2} + \frac{1}{2\pi}\oint_{0}^{\ell} \frac{g(\xi)}{x-\xi}\,d\xi$$

or, with the cavity boundary condition,

$$(18) \qquad \sigma = -\gamma(x) + \frac{1}{\pi}\oint_{0}^{\ell} \frac{g(\xi)}{x-\xi}\,d\xi$$

with

$$\gamma(x) = 0 \quad \text{for} \quad x \geq 1$$

The integral in equation (18) is a Cauchy principal value integral over the length of the cavity.

When equations (16) and (17) are subtracted from each other and the imaginary part is taken the result is:

$$v^{-}(x) = -\frac{g(x)}{2} + \frac{1}{2\pi}\oint_{0}^{1} \frac{\gamma(\xi)}{x-\xi}\,d\xi$$

or, with the wetted surface boundary condition,

$$(19) \qquad h_x^{-}(x) = -\frac{g(x)}{2} + \frac{1}{2\pi}\oint_{0}^{1} \frac{\gamma(\xi)}{x-\xi}\,d\xi$$

with

$$g(x) = 0 \quad \text{for} \quad x > \ell$$

Again the integral in equation (19) is a Cauchy principal value integral. For a flat plate at an angle of attack

$$(20) \qquad h_x^{-}(x) = -\alpha$$

and this is the case taken throughout the rest of this report. The source distribution q(x) represents the slope difference between the cavity surface and the mean camber line of the foil at x. While the vortex distribution $\gamma(x)$ represents the difference in the x component of perturbation velocity between the upper and lower surface at x.

$$(21) \quad q(x) = v^{+}(x) - v^{-}(x)$$

$$(22) \quad \gamma(x) = -u^{+}(x) + u^{-}(x)$$

These statements follow directly from equations (16) and (17). When equations (2), (7) and (22) are combined the jump in pressure on the foil is:

$$(23) \quad C_p^{+}(x) - C_p^{-}(x) = -2\gamma(x)$$

and the coefficient is:

$$(24) \quad C_L = -2\int_0^1 \gamma(x)\,dx$$

The combination of condition (C 3) and equation (27) gives, for the closure condition,

$$(25) \quad \int_0^l q(x)\,dx = 0$$

It is clear from equation (23) that condition (C 4) is:

$$(26) \quad \gamma(1) = 0$$

Equations (18), (19), (20), (25) and (26) constitute a complete problem statement for the steady flow cavitation of a two-dimensional flat plate, only, it is convenient to rearrange equations (18) and (19) into a form that contains the ratio of angle of attack to cavitation number.

$$(27) \qquad 1 = -\frac{\gamma(x)}{\sigma} + \frac{1}{\pi} \oint_0^l \frac{q(\xi)/\sigma}{x-\xi}\, d\xi$$

$$(28) \qquad 0 = \frac{\alpha}{\sigma} - \frac{1}{2}\frac{q(x)}{\sigma} + \frac{1}{2\pi} \oint_0^1 \frac{\gamma(\xi)/\sigma}{x-\xi}\, d\xi$$

and

$$(29) \qquad \int_0^l \frac{q(x)}{\sigma}\, dx = 0$$

Equations (27), (28) and (29), subject to $\gamma(1) = 0$, are sufficient to obtain cavity length, the source distribution and the vortex distribution as functions of the ratio α/σ only.

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The next section will develop the numerical approximation to equations (27), (28) and (29) and the method of solution.

CHAPTER IV

NUMERICAL APPROXIMATION TO THE INTEGRAL EQUATIONS

The method used in the numerical approximation is to approximate the integral equations (27), (28) and (29) by a finite set of linear algebraic equations with N unknown source densities, M unknown vortex densities and the ratio α/σ . The numerical integration procedure requires that cavity length be the independent variable and $\frac{\alpha}{\sigma}$ be computed.

This method requires an assumption about the functional form of the source and vortex distributions. It is assumed that the vortex and source distributions are constant over a small element of the foil or cavity, figure 2. The last vortex element is assumed to be linear with a value of zero at the trailing edge to satisfy equation (26). The amplitude of each γ_i , q_i is unknown.

In each vortex element a control point is placed at which the flow tangency boundary condition is satisfied. In each source element a control point is placed at which the cavity boundary condition is satisfied. The vortex control points are at x_j and the source control points are at x_{sj} . The relations for the control point locations are:

1) for the vortex control

$$x_j = x_{\ell j} + (x_{\ell j+1} - x_{\ell j}) f , \qquad 0 < f < 1 \tag{30}$$

[illegible]

and

2) for the sources

(31) $$x_{s_j} = x_{\ell j} + (x_{\ell j+1} - x_{\ell j}) f_s \qquad 0 < f_s < 1$$

The fractions f and f_s are yet to be determined on the basis of obtaining stable results.

There will be one equation for each vortex control point, giving M equations. There will be one equation for each source control point, giving N equations. Plus there will be one equation for the cavity closure condition. The vortex equations are:

(32) $$0 = 2\pi \frac{\alpha}{\sigma} - \pi \frac{q_j}{\sigma} + \oint_{x_{\ell j}}^{x_{\ell j+1}} \frac{\gamma_j/\sigma}{x_j - \xi} d\xi + \sum_{\substack{i=1 \\ i \neq j}}^{M-1} \int_{x_{\ell i}}^{x_{\ell i+1}} \frac{\gamma_i/\sigma}{x_j - \xi} d\xi + \int_{x_{\ell M}}^{1} \frac{\frac{\gamma_M}{\sigma}(1-\xi)}{(x_j - \xi)(1 - x_{\ell M})} d\xi$$

and for the Mth control point

(33) $$0 = 2\pi \frac{\alpha}{\sigma} - \frac{q_M}{\sigma} + \oint_{x_{\ell M}}^{1} \frac{\frac{\gamma_M}{\sigma}(1-\xi)}{(x_M - \xi)(1 - x_{\ell M})} d\xi + \sum_{i=1}^{M-1} \int_{x_{\ell i}}^{x_{\ell i+1}} \frac{\gamma_i/\sigma}{x_M - \xi} d\xi$$

Where,

$$q_j/\sigma = 0 \text{ for } j > N \quad \text{and} \quad q_M/\sigma = 0 \text{ for } M > N$$

The source equations are:

$$(34)\qquad \pi = -\pi\frac{\gamma_j}{\sigma} + \sum_{\substack{i=1\\ i\neq j}}^{N}\int_{x_{\ell i}}^{x_{\ell i+1}} \frac{q_i/\sigma}{x_j-\xi}\,d\xi \; +$$

$$+ \oint_{x_{\ell j}}^{x_{\ell j+1}} \frac{q_j/\sigma}{x_j-\xi}\,d\xi \quad , \quad \gamma_j = 0 \text{ for } j > M$$

and the closure condition

$$(35)\qquad \sum_{i=1}^{N}\int_{x_{\ell i}}^{x_{\ell i+1}} \frac{q_i}{\sigma}\,dx = 0$$

Equations (32) through (35) are the numerical approximation to the integral equations. When the indicated integration is performed equations (32), (33), (34) and (35) become

$$(36)\qquad 0 = 2\pi\frac{\alpha}{\sigma} - \pi\frac{q_j}{\sigma} + \frac{\gamma_j}{\sigma}\ln\left(\frac{f}{1-f}\right) + \sum_{\substack{i=1\\ i\neq j}}^{M-1}\frac{\gamma_i}{\sigma}\ln\left(\frac{x_j-x_{\ell i}}{x_j-x_{\ell i+1}}\right) +$$

$$+ \frac{\gamma_M}{\sigma}\left[\frac{1-x_j}{1-x_{\ell M}}\ln\left(\frac{x_j-x_{\ell M}}{x_j-1}\right) + 1\right] \quad , \quad \frac{q_j}{\sigma} = 0 \text{ for } j>N$$

$$(37)\qquad 0 = 2\pi\frac{\alpha}{\sigma} - \pi\frac{q_M}{\sigma} + \frac{\gamma_M}{\sigma}\left[1 + (1-f)\ln\left(\frac{f}{1-f}\right)\right] +$$

$$+ \sum_{i=1}^{M-1}\frac{\gamma_i}{\sigma}\ln\left(\frac{x_M-x_{\ell i}}{x_M-x_{\ell i+1}}\right) \quad , \quad \frac{q_M}{\sigma} = 0 \text{ for } M > N$$

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$$(38) \qquad \pi = -\pi\frac{\gamma_j}{\sigma} + \frac{q_j}{\sigma}\ln\left(\frac{f_s}{1-f_s}\right) + \sum_{\substack{i=1\\ i\neq j}}^{N}\frac{q_i}{\sigma}\ln\left(\frac{x_j - x_{\ell i}}{x_j - x_{\ell i+1}}\right)$$

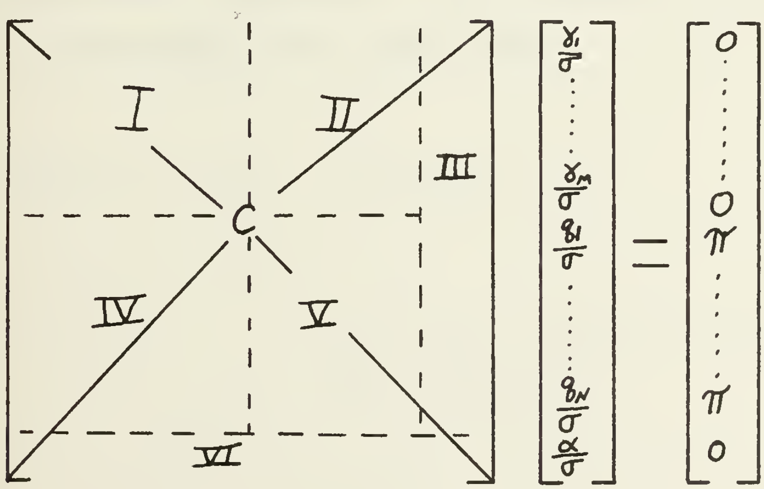
$$\frac{\gamma_j}{\sigma} = 0 \quad \text{for } j > N$$

and

$$(39) \qquad \sum_{i=1}^{N}\frac{q_i}{\sigma}\left(x_{\ell i+1} - x_{\ell i}\right) = 0$$

Equations (36) through (39) are linear algebraic equations in the unknowns $\frac{\alpha}{\sigma}$, $\frac{q_i}{\sigma}$ and $\frac{\gamma_i}{\sigma}$. It is convenient to rewrite these equations in matrix form.

(40)



The coefficient matrix, [C], is formed from the coefficients of $\frac{\alpha}{\sigma}$, $\frac{q_i}{\sigma}$, $\frac{\gamma_i}{\sigma}$, $\frac{q_j}{\sigma}$ and $\frac{\gamma_j}{\sigma}$ in equations (36) through (39).

The first submatrix, (I), is formed as follows by the coefficients of γ/σ in equations (36) and (37).

(41) $$C_{jj} = \ln\left(\frac{f}{1-f}\right), \qquad j \neq M$$

(42) $$C_{MM} = 1 + (1-f)\ln\left(\frac{f}{1-f}\right)$$

(43) $$C_{ji} = \ln\left(\frac{x_j - x_{\ell i}}{x_j - x_{\ell i+1}}\right), \qquad i \neq M \text{ and } j \neq M$$

(44) $$C_{jM} = \frac{1-x_j}{1-x_{\ell M}} \ln\left(\frac{x_j - x_{\ell M}}{x_j - 1.}\right) + 1, \qquad j \neq M$$

The second submatrix, (II), is given by the coefficient of q_j/σ in equations (36) and (37):

(45) $$C_{j,j+M} = -\eta, \qquad 1 \leq j \leq N$$

(46) $$C_{ji} = 0, \qquad 1 \leq j \leq M, \; M+1 \leq i \leq M+N \text{ and } i \neq j+M.$$

Submatrix (III) is the coefficient of α/σ in equations (36) and (37):

$$(47) \qquad C_{j,M+N+1} = 2\pi \qquad \text{for} \quad 1 \le j \le M$$

$$(48) \qquad C_{j,M+N+1} = 0 \qquad \text{for} \quad M < j \le M+N+1 .$$

The source equations are contained submatricies (IV), (V), and (VI). The submatrix (IV) contains the coefficients of γ_i/σ in equation (38) :

$$(49) \qquad C_{M+j,j} = -\pi , \qquad 1 \le j \le M$$

and

$$(50) \qquad C_{ji} = 0 \qquad \text{otherwise} .$$

Submatrix (V) contains the coefficients of $\frac{q}{\sigma}$ in equation (38):

$$C_{M+j,M+i} = \ln\left(\frac{x_j - x_{\ell i}}{x_j - x_{\ell i+1}}\right), \qquad \begin{matrix} M+j \neq M+i \\ 1 \leq j, i \leq N \end{matrix} \tag{51}$$

and

$$C_{M+j,M+j} = \ln\left(\frac{f_s}{1-f_s}\right) \tag{52}$$

The last submatrix (VI), contains the closure condition, equation (39):

$$\begin{aligned} C_{M+N+1,i} &= x_{\ell i+1} - x_{\ell i}, & M+1 \leq i \leq M+N \\ C_{M+N+1,i} &= 0 & \text{otherwise} \end{aligned} \tag{53}$$

To solve equation (40) for the unknown vortex and source distributions and the ratio α/σ the coefficient matrix is inverted. The inverted coefficient matrix is then multiplied by the boundary condition matrix.

Thus the solution can be written as:

$$\begin{bmatrix} \frac{\gamma_1}{\sigma} \\ \vdots \\ \frac{\gamma_M}{\sigma} \\ q_1/\sigma \\ \vdots \\ q_N/\sigma \\ \frac{\alpha}{\sigma} \end{bmatrix} = \begin{bmatrix} \\ & C^{-1} & \\ \\ \end{bmatrix} \begin{bmatrix} 0 \\ \vdots \\ 0 \\ \pi \\ \vdots \\ \pi \\ 0 \end{bmatrix} \tag{54}$$

On the surface this procedure appears to be straightforward and should produce a solution without difficulty. However, this is not the case.

Difficulties in obtaining a meaningful numerical solution lie in the selection of control point locations within each source or vortex element. Control point locations are critical to approximating the singular behavior of the analytic solution. It was found that vortex control points should be at 90 percent of the element length and source control points at the element midpoint. However, special care must be taken with a few of the source control points.

The critical control points are the first and last source control points. A systematic variation of the control point position determined that the last source control point should be placed at 10 percent of the element length. As an example of the importance of this control point, the variation of the computed ratio of $\frac{\alpha}{\sigma}$ with control point location is shown in figure 3 for a cavity 0.5 times chord length.

The selected control point position is at 10 percent of the element length. Variation of the first source control point indicated the placement should be at 90 percent of the element length. Since the source distribution is singular at the cavity leading edge and termination, these control points should be placed away from the singularities.

Variation of other control points (second and next to last source, last vortex, first source past trailing edge for super cavity) showed the best placement to be the generalized locations. Thus the fractions f and f_s are:

$f = 0.90$ for all vortex elements,

$f_s = 0.90$ for first source control point,

$f_s = 0.10$ for last source control point,

and

$f_s = 0.50$ for all other source control points.

The computer program that executes the solution to the described numerical procedure is listed in Appendix A. This program is composed of a main program, which performs Input/Output operations and logic control, and three subroutines.

The first subroutine, CPGEN, determines the vortex and source element sizes and the locations of the control points. The required input information for this subroutine is the cavity length, the number of vortex elements and the number of source elements.

The second subroutine, MATRIX, computes the coefficient matrix, $[C]$, and the boundary condition vector. The required input information for MATRIX is the output of CPGEN and the number of vortex and source elements.

The last subroutine, RMINV, inverts the coefficient matrix. This subroutine is a standard MIT matrix inversion routine [11]. The inverted matrix is then multiplied by the

boundary condition vector, in the main program, to obtain the solution vector for $\frac{\alpha}{\sigma}$, $\frac{\gamma_i}{\sigma}$ and $\frac{q_i}{\sigma}$.

A special note on the program notation is made here. The vector XU(I) is the upper boundary of a vortex or source element. This vector is given by:

$$XU(I) \equiv XL(I + 1)$$

through an assignment statement in CPGEN.

CHAPTER V

ANALYTIC SOLUTION

The method of gauging the numerical results is the analytic solution for the cavitating flow of two-dimensional flat plate. The first half of this chapter will discuss Geurst's Solution for partial cavitation [1,2] . The second half will discuss Geurst's solution for super cavitation [3] .

Geurst solves the partial cavitation problem by a conformal mapping of the z plane to the ζ plane as follows. First the z plane is given by:

$$(55) \qquad z = x + iy , \qquad x = \begin{cases} -1 & \text{corresponds to leading edge} \\ +1 & \text{corresponds to trailing edge} \end{cases}$$

The mapping function is:

$$(56) \qquad \zeta = \sqrt{\frac{1+z}{1-z}} , \qquad \zeta = \xi + i\eta$$

This mapping is such that the point at infinity in the physical plane is mapped to $\zeta = i$. The flow is then in the upper half of the ζ plane. Figure 4(a) shows the resulting plane.

The solution for the complex velocity is:

$$(57) \qquad \frac{w(\zeta)}{\sigma} = \frac{\frac{A}{\sigma}\zeta + \frac{B}{\sigma}}{\sqrt{\zeta(\zeta - b)}}$$

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

where,

$$b = \sqrt{\frac{1+\bar{\ell}}{1-\bar{\ell}}}$$

$$\ell = \frac{1+\bar{\ell}}{2} = \cos^2\delta$$

$$\frac{A}{\sigma} = \left[\tan\delta \, \frac{1-\sin\delta}{1+\sin\delta} \sin\left(\frac{\pi}{4}-\frac{\delta}{2}\right) - \cos\left(\frac{\pi}{4}-\frac{\delta}{2}\right)\right]\left[\frac{1}{2\sqrt{\sin\delta}}\right]$$

and

$$\frac{B}{\sigma} = \left[\tan\delta \, \frac{1-\sin\delta}{1+\sin\delta} \cos\left(\frac{\pi}{4}-\frac{\delta}{2}\right) + \sin\left(\frac{\pi}{4}-\frac{\delta}{2}\right)\right]\left[\frac{1}{2\sqrt{\sin\delta}}\right]$$

With equation (57) is substituted into equations (21) and (22) over the cavity and wetted surface of the foil, the vortex and source distributions can be obtained. Figures 5 and 6 show the vortex and source distributions for cavity equal to one-half chord length. These figures show the leading edge and cavity termination singularities. One of the difficulties for the numerical procedure is to reasonably approximate these four singularities.

Other quantities of interest are the ratio $\frac{\alpha}{\sigma}$, the lift coefficient and the cavity area. These are given as:

(58) $$\frac{\alpha}{\sigma} = \frac{1}{2} \tan\delta \left[\frac{1-\sin\delta}{1+\sin\delta}\right]$$

(59) $$\frac{C_L}{2\pi\alpha} = \frac{1}{2}\left[1 + \frac{1}{\sin\delta}\right]$$

and

(60) $$\frac{Area}{2\pi\alpha} = \frac{1}{16}\Big\{(1+\sin\delta)(-1+3\sin\delta)\cos\delta\sin\delta + \frac{1}{2}\cot an\,\delta(1+\sin\delta)(1+3\sin\delta-2\sin^2\delta-6\sin^3\delta)\Big\}$$

The equation for cavity area given in [1] is in error and was corrected to equation (60) in [2]. Plots of equations (58) through (60) are shown in figures 7, 8 and 9.

To obtain a solution to the super cavitating problem, Geurst performs a similar mapping (figure 4(b)) defined by:

$$(61) \qquad \zeta = \sqrt{\frac{z}{\ell - z}}$$

The solution for the complex velocity is then.

$$(62) \qquad \frac{w(\zeta)}{\sigma} = \frac{\frac{A}{\sigma}\zeta + \frac{B}{\sigma}}{i} \sqrt{\frac{\zeta + b}{\zeta}}$$

where

$$b = \frac{1}{\ell - 1} = \cot\delta$$

$$\frac{1}{\ell} = \cos^2\delta$$

$$\frac{A}{\sigma} = \frac{\sqrt{\sin\delta}}{2}\left[\tan\delta \sin\left(\frac{\pi}{4} - \frac{\delta}{2}\right) - \cos\left(\frac{\pi}{4} - \frac{\delta}{2}\right)\right]$$

and

$$\frac{B}{\sigma} = \frac{\sqrt{\sin\delta}}{2}\left[\tan\delta \cos\left(\frac{\pi}{4} - \frac{\delta}{2}\right) + \sin\left(\frac{\pi}{4} - \frac{\delta}{2}\right)\right]$$

The vortex and source distributions are obtained using the same procedure as with the partial cavitation case. Figures 10 and 11 depict these distributions. In the super cavitation case only a leading edge singularity appears in the vortex distribution. Again the source distribution shows the leading edge and cavity termination singularities.

The quantities $\frac{\alpha}{\sigma}$, lift coefficient and cavity area are:

$$\frac{\alpha}{\sigma} = \frac{1}{2} \tan \delta \tag{63}$$

$$\frac{C_L}{2\pi\alpha} = \frac{1}{2 \sin \delta (1 + \sin \delta)} \tag{64}$$

and

$$\frac{\text{Area}}{2\pi\alpha} = \frac{\cos \delta}{32 \sin \delta (1 - \sin \delta)^2} \tag{65}$$

Figures 12, 13 and 14 depict equations (63) through (65). As with partial cavitation when the cavity approaches chord length the lift coefficient and cavity area become singular.

In chapter VI the computed results are compared to the analytic solution.

CHAPTER VI

RESULTS, CONCLUSIONS AND RECOMMENDATIONS

The primary result of this investigation is a numerical method for two-dimensional cavitating flows which gives computed values close to the analytic solution. The computed results have been plotted in figures 3 and 5 through 16. In figures 5 through 14, the analytic results of Geurst are also shown.

Figures 5 and 6 show the computed vortex and source distributions for a cavity of one-half chord length. Each computed point is plotted at the control point location, except for the last vortex element which is plotted at $x_{\ell M}$. It is to be remembered that each computed point represents a constant value from $x_{\ell i}$ to $x_{\ell i+1}$. These two figures show that the computed distributions fit the analytic solution reasonably well. However, the largest magnitude elements are not shown on the figures (see the sample computer output in Appendix B for their values). These elements do not appear to approximate the proper singular behavior.

Near the singularities the distributions of sources and vorticies can be approximated by $1/x^n$. For the leading edge n has the value one-quarter and for cavity termination n has the value one-half[1]. The computed values of n are given in Table 4 for three values of M and N. From Table 4 it is apparent that the behavior of the two closest elements does not match closely the analytic solution's singular behavior.

Figures 7, 8 and 9 compare the computed values of α/σ, $C_L/2\pi\alpha$ and $A/2\pi\alpha$, for ten cavity lengths to the analytic solution. The plotted results show the points closest to the analytic solution from a number of computations for various values of M and N. In general the computed results compare quite favorably with the analytic solution. The exception to this is for cavity lengths approaching chord length. For these cavities the method appears to be very slowly convergent. For these cavities a large number of elements are required for the cavity-foil region leading to a large number of equations.

Table 3 contains all the computed results to date. This table gives results for variations of M and N for specific cavity lengths. For example, taking $\ell = 0.5$, the results show that when the vortex and source element size is smaller in the combined cavity-foil region than in the fully wetted region, the computed values show an improved level of convergence to the analytic solution. This result is, also shown in figures 5, 6, and 15.

Figure 15 shows the results of a convergence test for a one-half chord length cavity . this figure also shows the tendancy for better convergence for a smaller interior element size. It is noted that for figure 15 the vortex element size in the region of fully wetted flow are the same for both ratios of M and N. This effect is also shown in figure 5 for the vortex distribution with the same ratios of M to N and the source distribution in figure 6.

This conclusion points out one of the problems with the numerical method used herein. The method reported on requires that the vortex and source elements in the combined cavity-foil region be of equal size. An independent variation of the source and vortex element sizes cannot be performed. Thus it is not possible to determine whether decreasing the size of the vortex elements, the source elements,or both, results in the improved convergence. It is recommended that an investigation of effects of independent variations of the vortex and source elements be done. This can be achieved through a moderate revision of the computer program.

Also, since the singularities do not appear to be well matched, a further investigation of the location of control points should be considered. It seems desirable to have an analytic basis for the control point locations.

From the general form of the computed vortex and source distributions, this investigator is convinced that assuming piecewise linear distributions will produce an immediate improvement in the computational results. It is, therefore, recommended that this numerical method be modified to incorporate piecewise linear vortex and source distributions.

The results for the super cavitation case are very similar to the partial cavitation case. The convergence to the analytic solution becomes worse as the cavity length approach chord length. The same conclusions and recommendations made for the partial cavity apply to the super cavity.

[illegible]

Another purpose of this investigation has been to develop a method of allowing arbitrary values of angle of attack and camber as inputs and then determine the cavity length. An itterative procedure to accomplish this has been developed. The method and the computer program are in Appendix C with the computed results. The method and program are a preliminary effort provided to prove only that such a method is possible.

In summary these are five recommendations for future investigation.

1) A program be written that allows the vortex and source element sizes to be determined independently.

2) An analytic and further numerical investigation of control point locations.

3) Piecewise linear distributions of sources and vorticies should be investigated.

4) That the method herein not be applied to the three-dimensional case without further investigation.

5) The program should be modified to include cambered profiles. In this case the ratio of α/σ cannot be computed as an unknown. Either the angle of attack or cavitation number must be known along with the camber line. It is not clear that the control point locations will be the same as with the flat plate.

6) Further development of the method of computing cavity length from arbitrary values of α and σ .

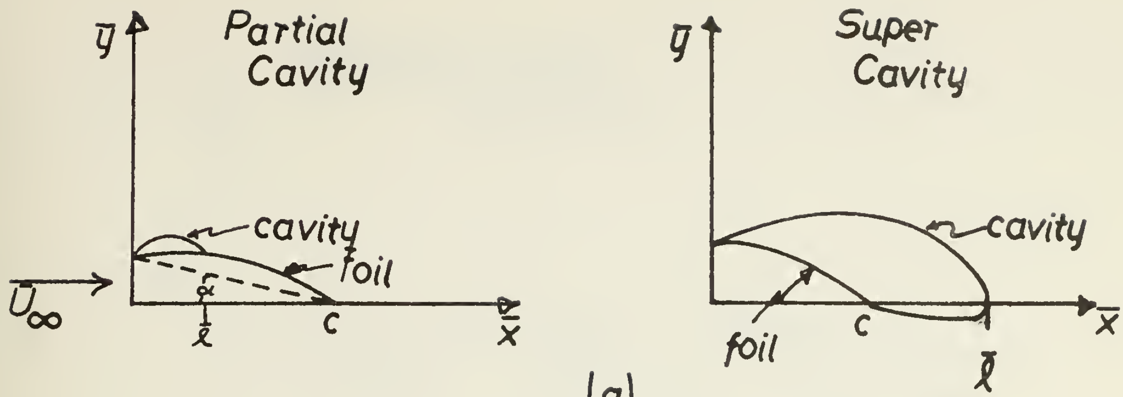
The conclusions drawn from this investigation are:

1) the method gives computed values close to the analytic solution,

2) the distribution of sources and vorticies is probably the best test of computed results,

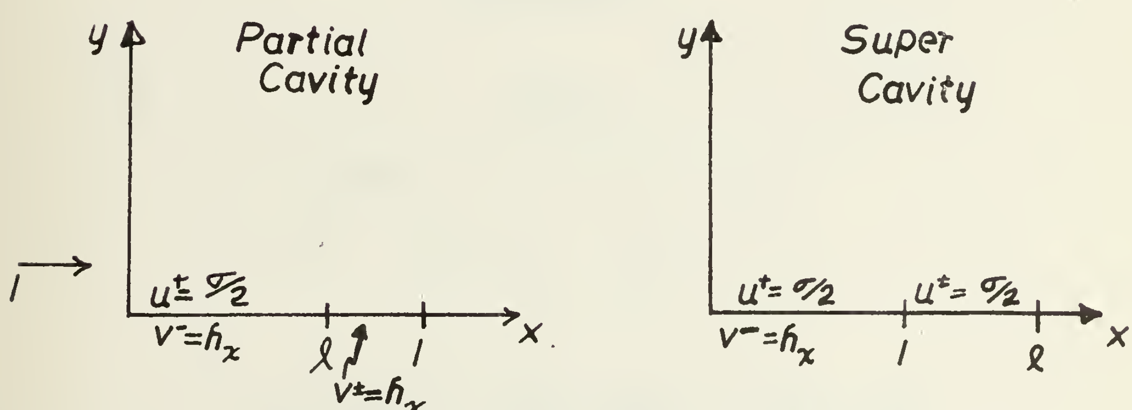
3) the method shows poor convergence for cavity lengths near chord length,

4) an iterative procedure to determine cavity length from an input of angle of attack and cavitation number is possible.



(a)

Dimensional



(b)

Nondimensional - Linear

figure 1
Coordinate Systems

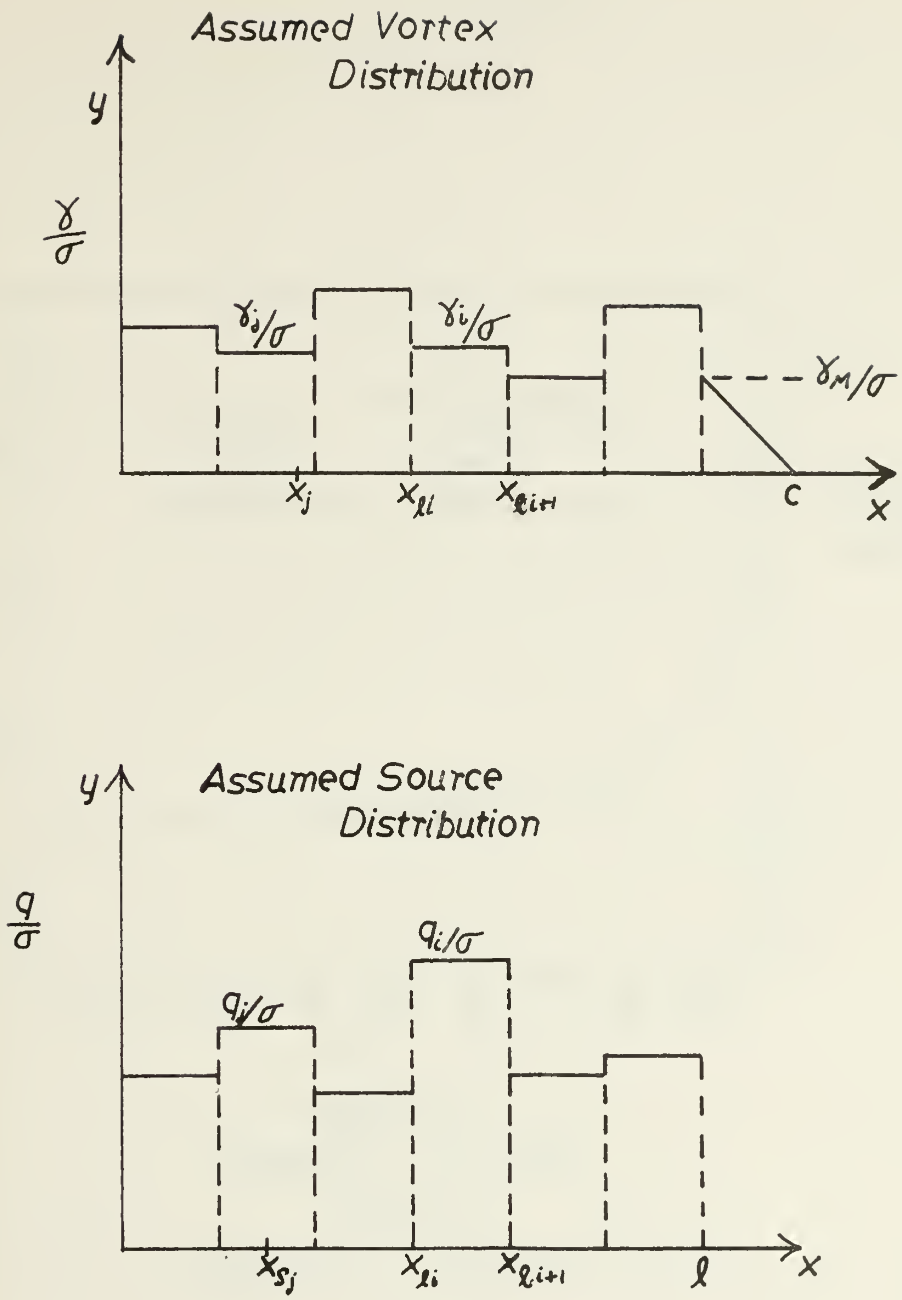


figure 2
Element Arrangements

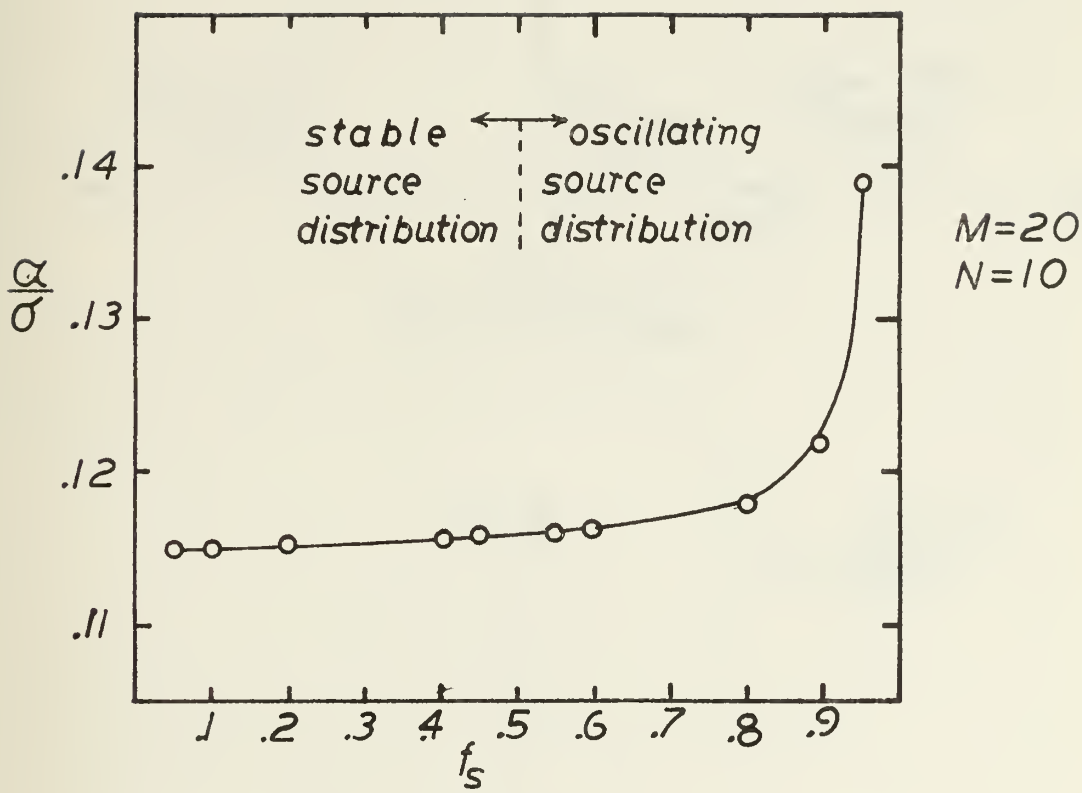


figure 3
Variation of Last
Source
Control Point for $\ell=0.5$

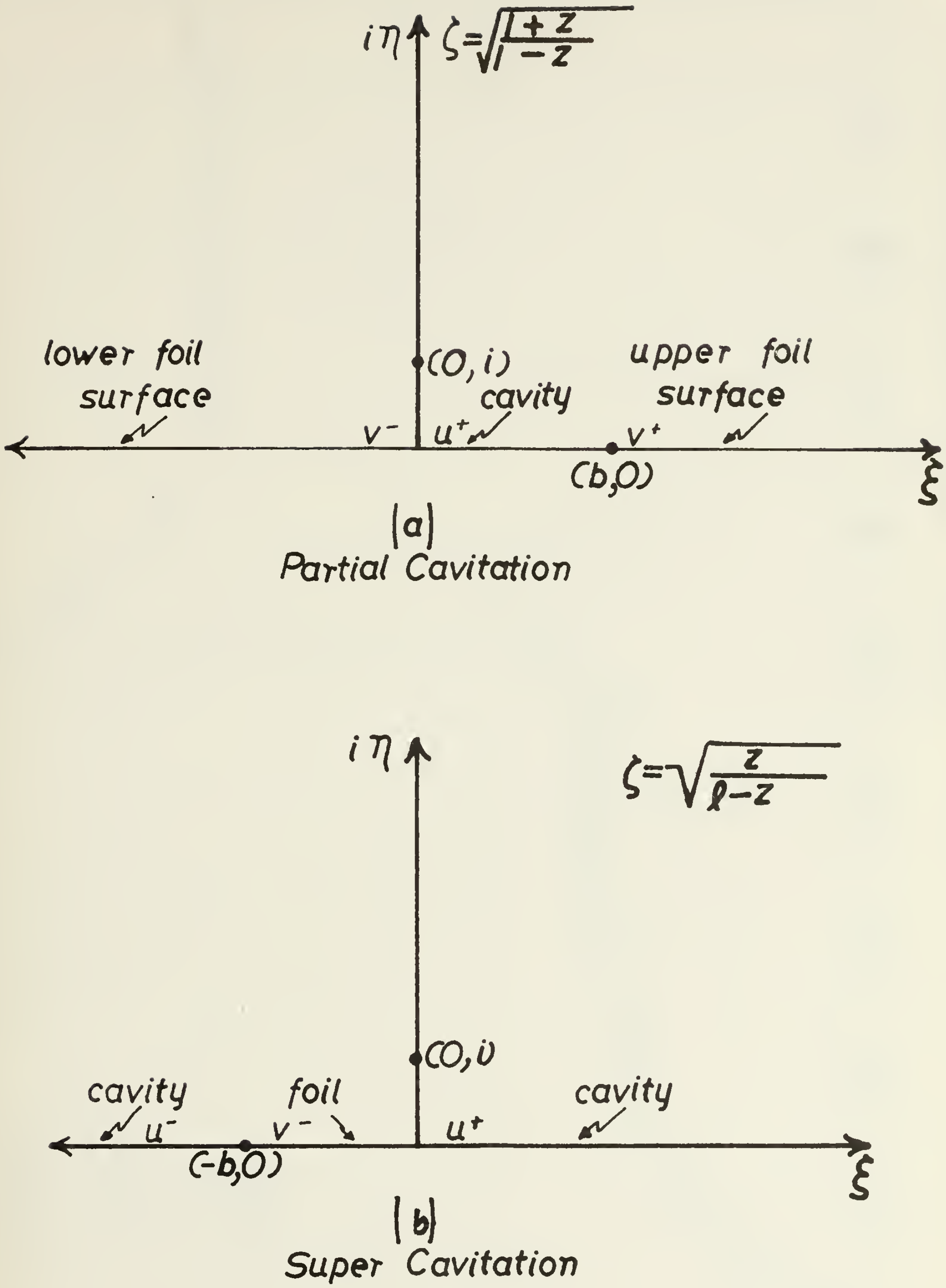


figure 4
Mapped Planes

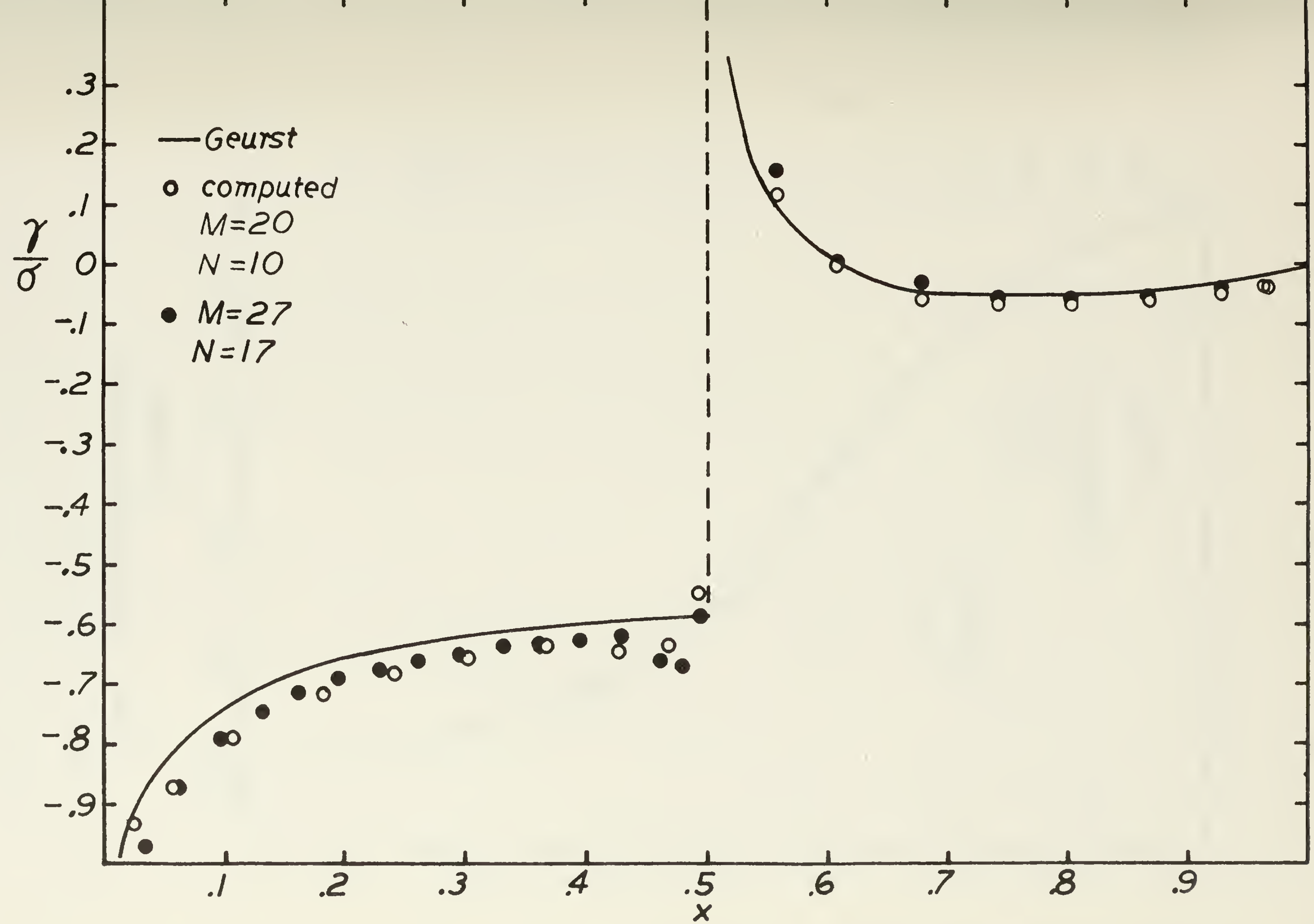


figure 5 Vortex Distribution, $\ell=0.5$

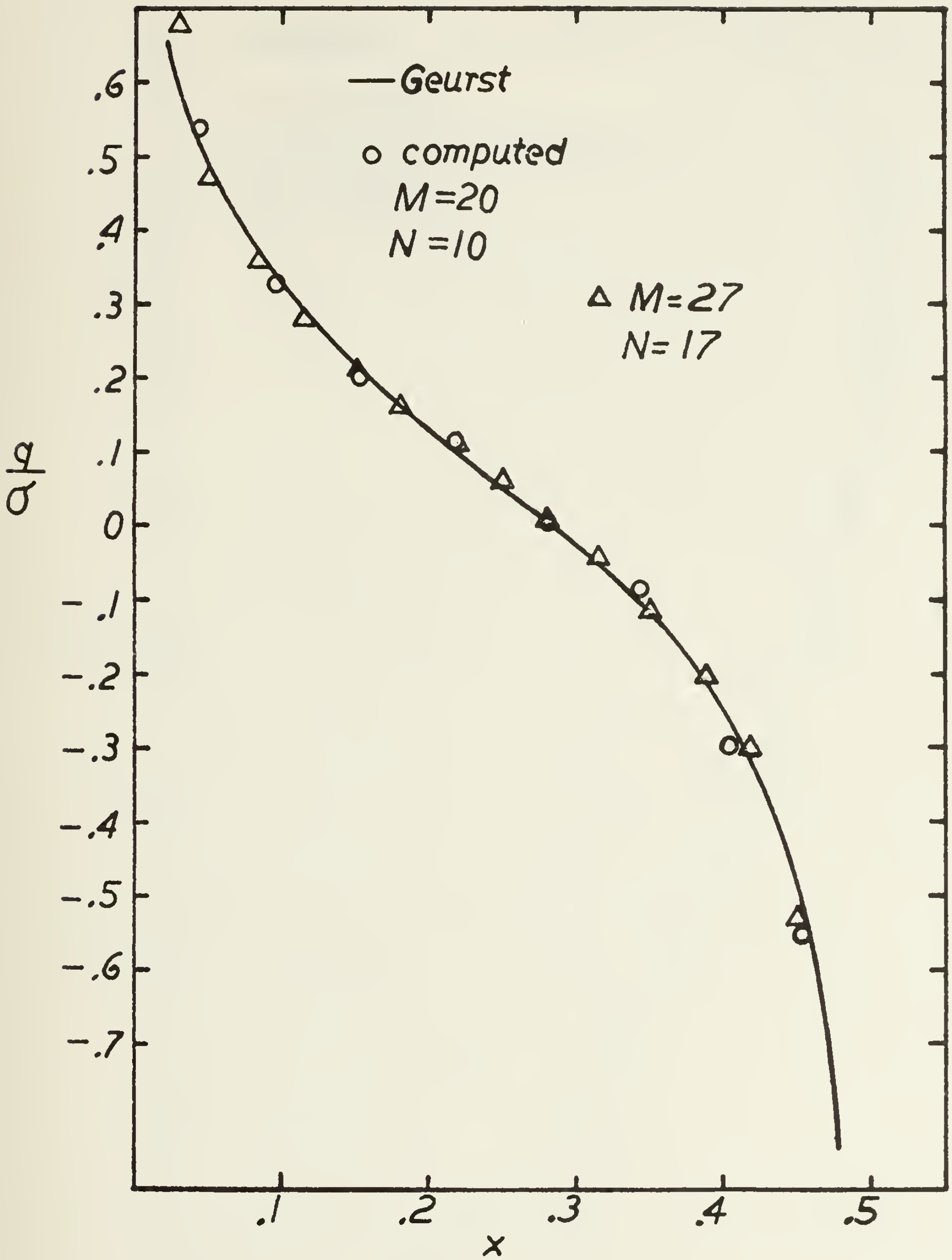


figure 6
Source Distribution, ℓ=0.5

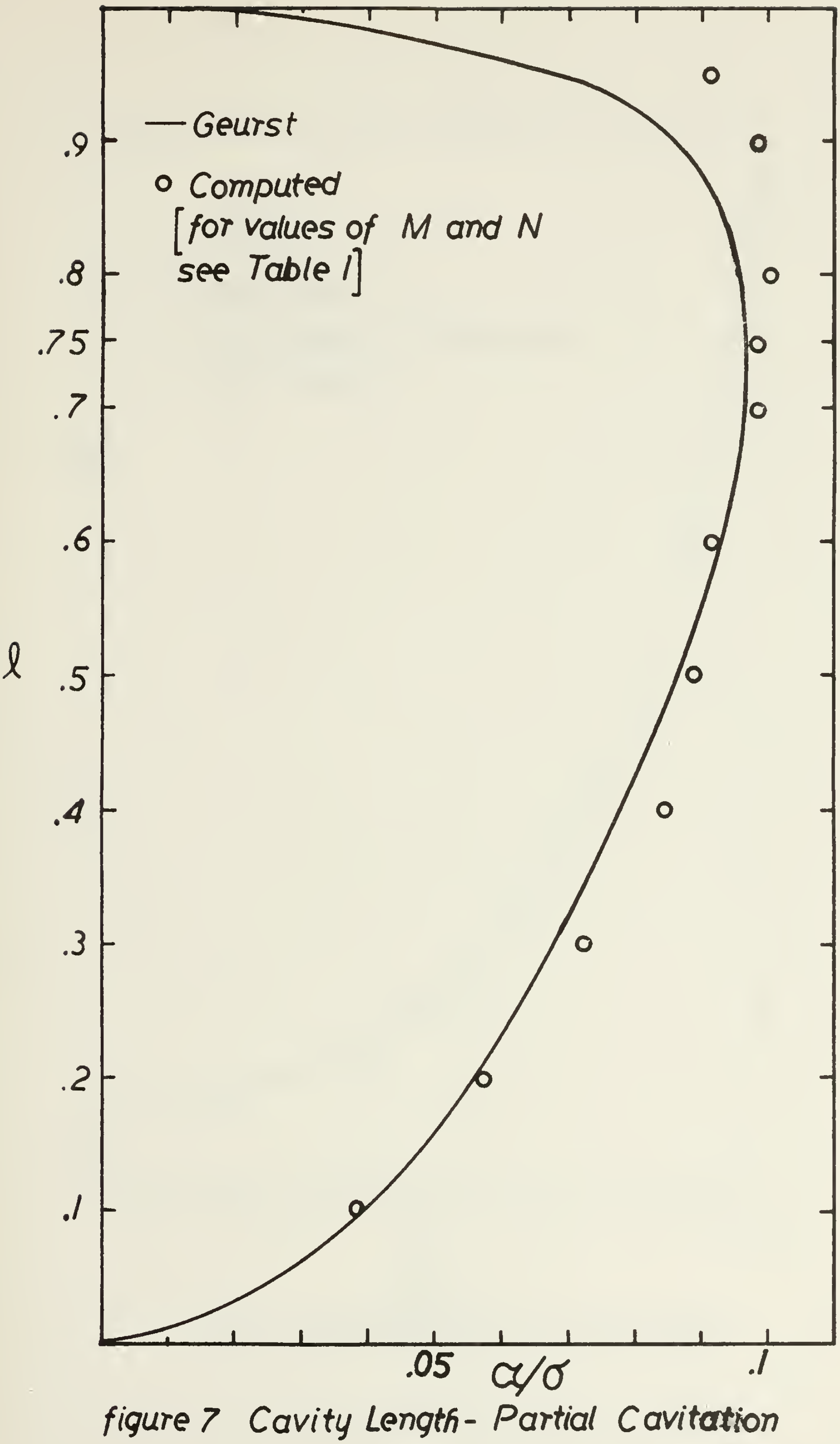


figure 7 Cavity Length - Partial Cavitation

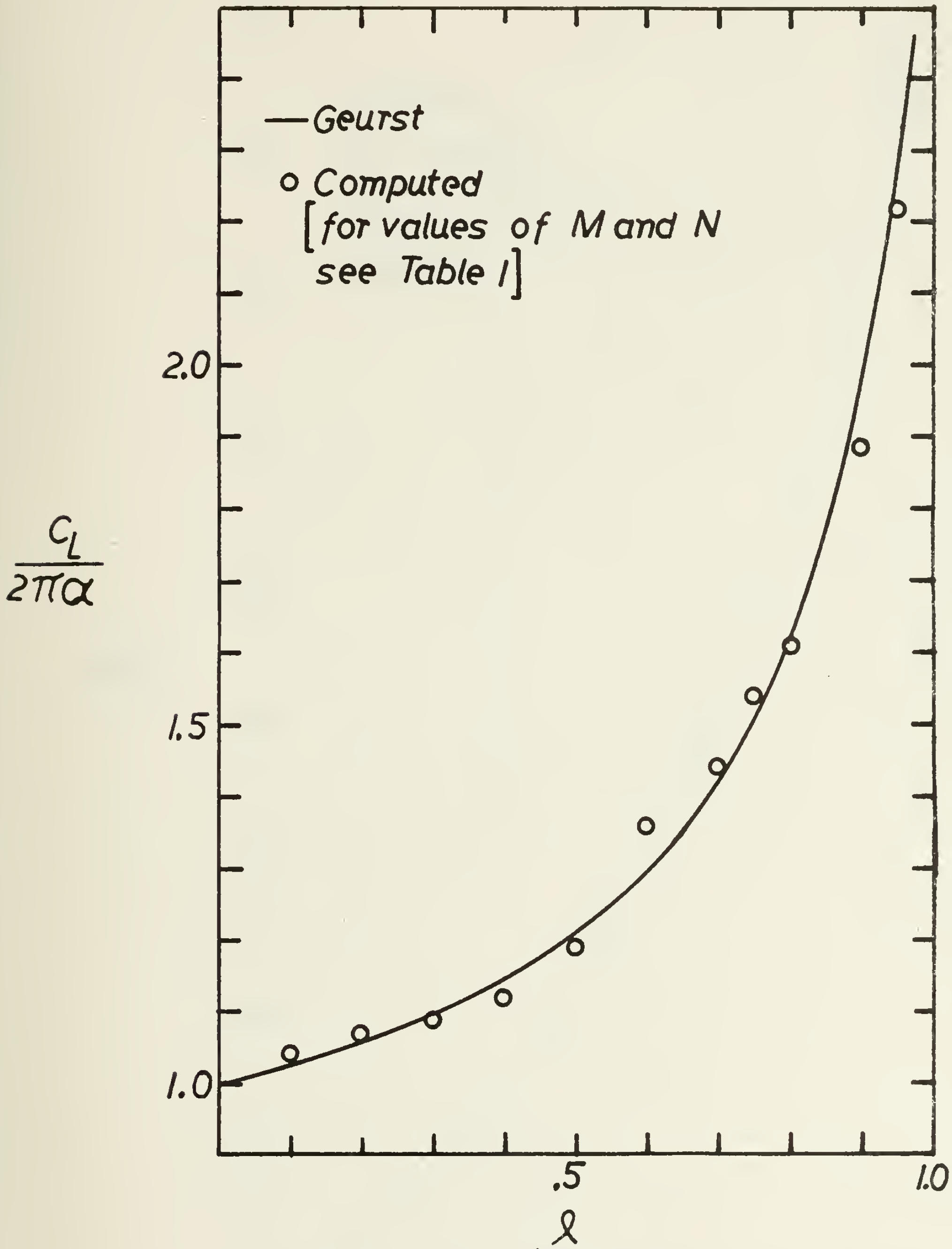


figure 8
Lift Coefficient - Partial Cavitation

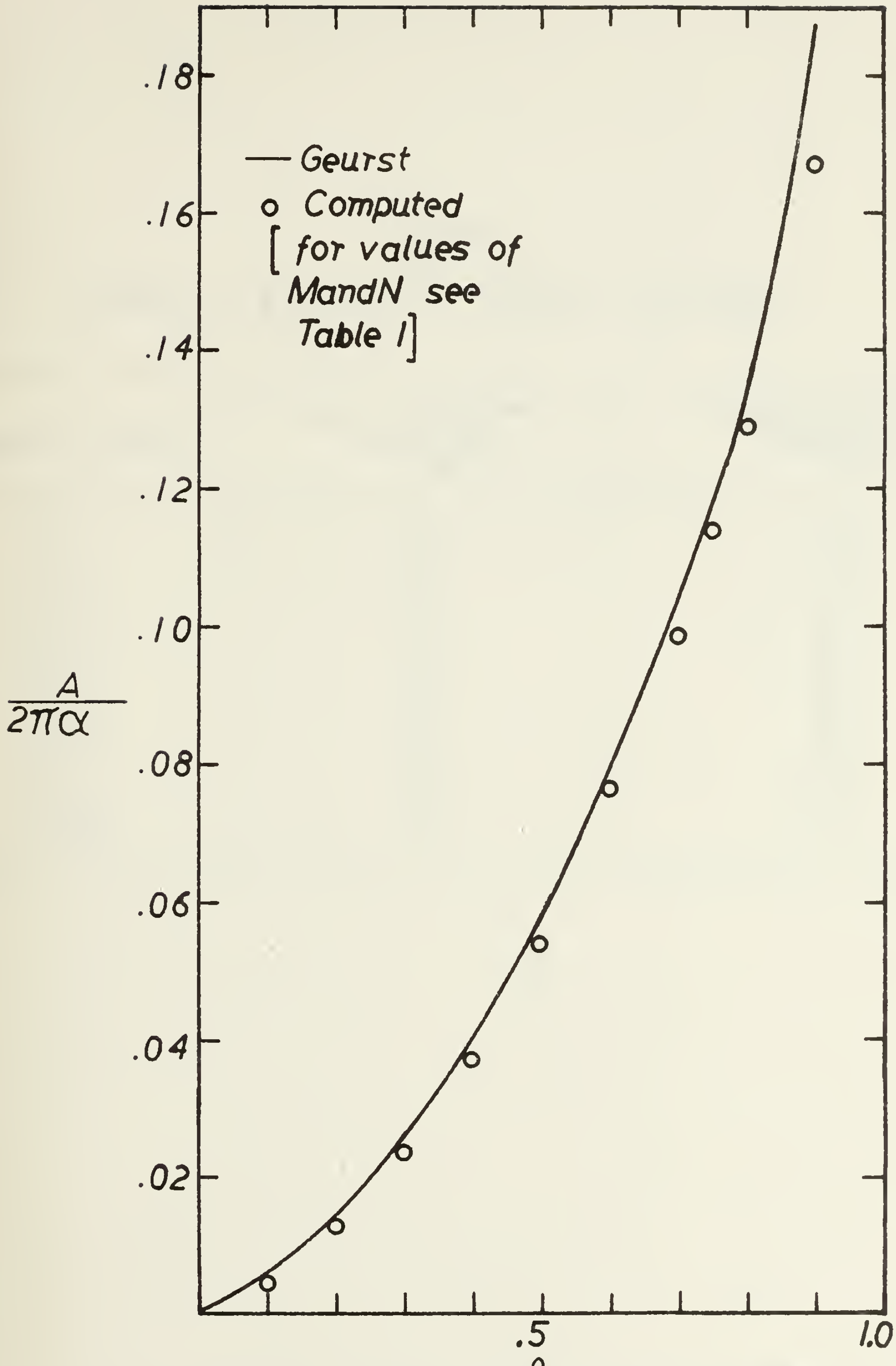


figure 9 Cavity Area — Partial Cavitation

Table 1

Values of M and N for figures 5 through 9

Cavity length ℓ	No. of Vortex elements M	No. of Source elements N
0.1	30	15
0.2	30	15
0.3	30	15
0.4	30	15
0.5	54	34
0.6	25	20
0.7	25	20
0.75	30	25
0.8	30	25
0.9	30	25
0.95	30	25

[illegible]

[illegible]

[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]

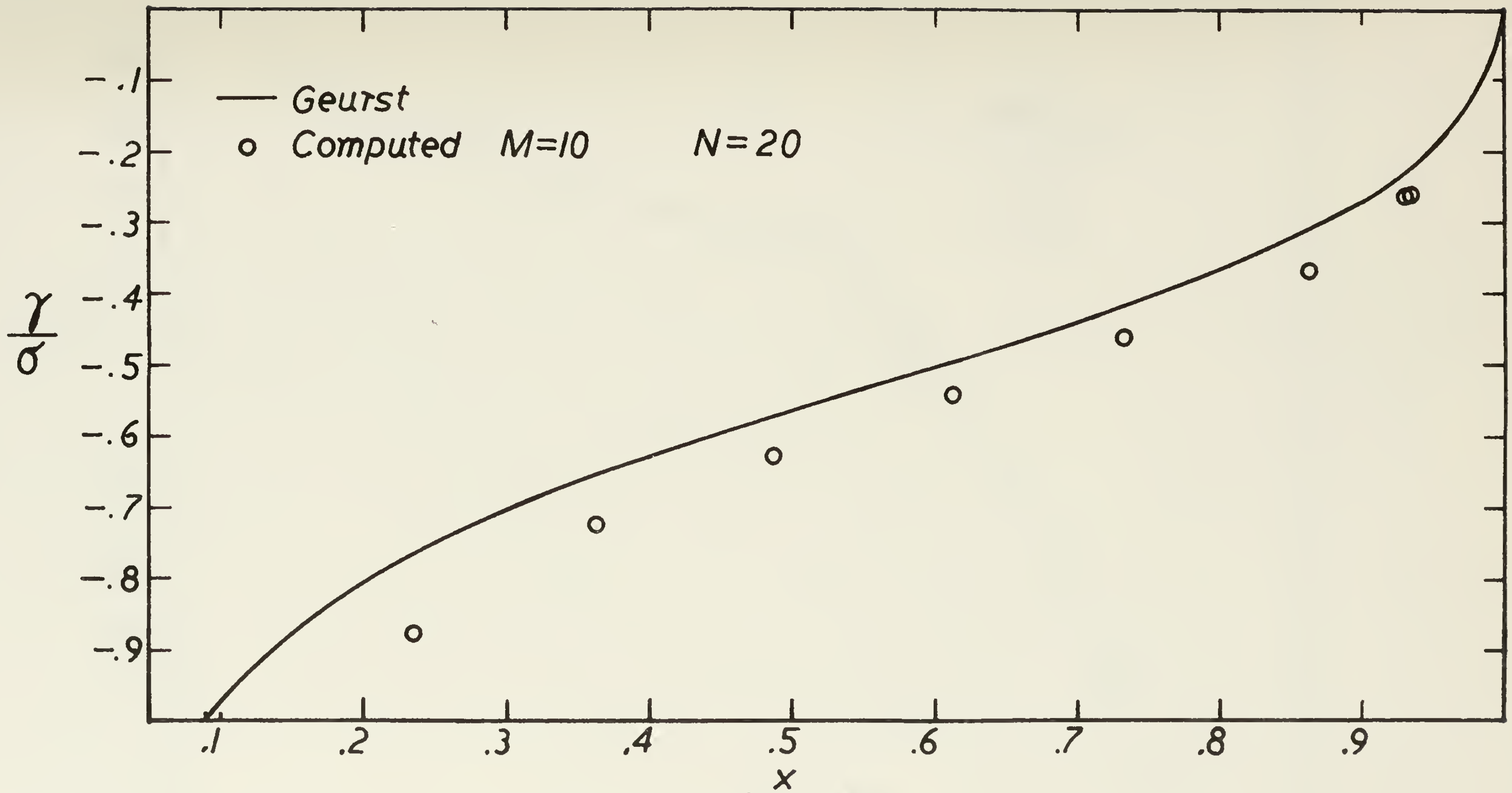
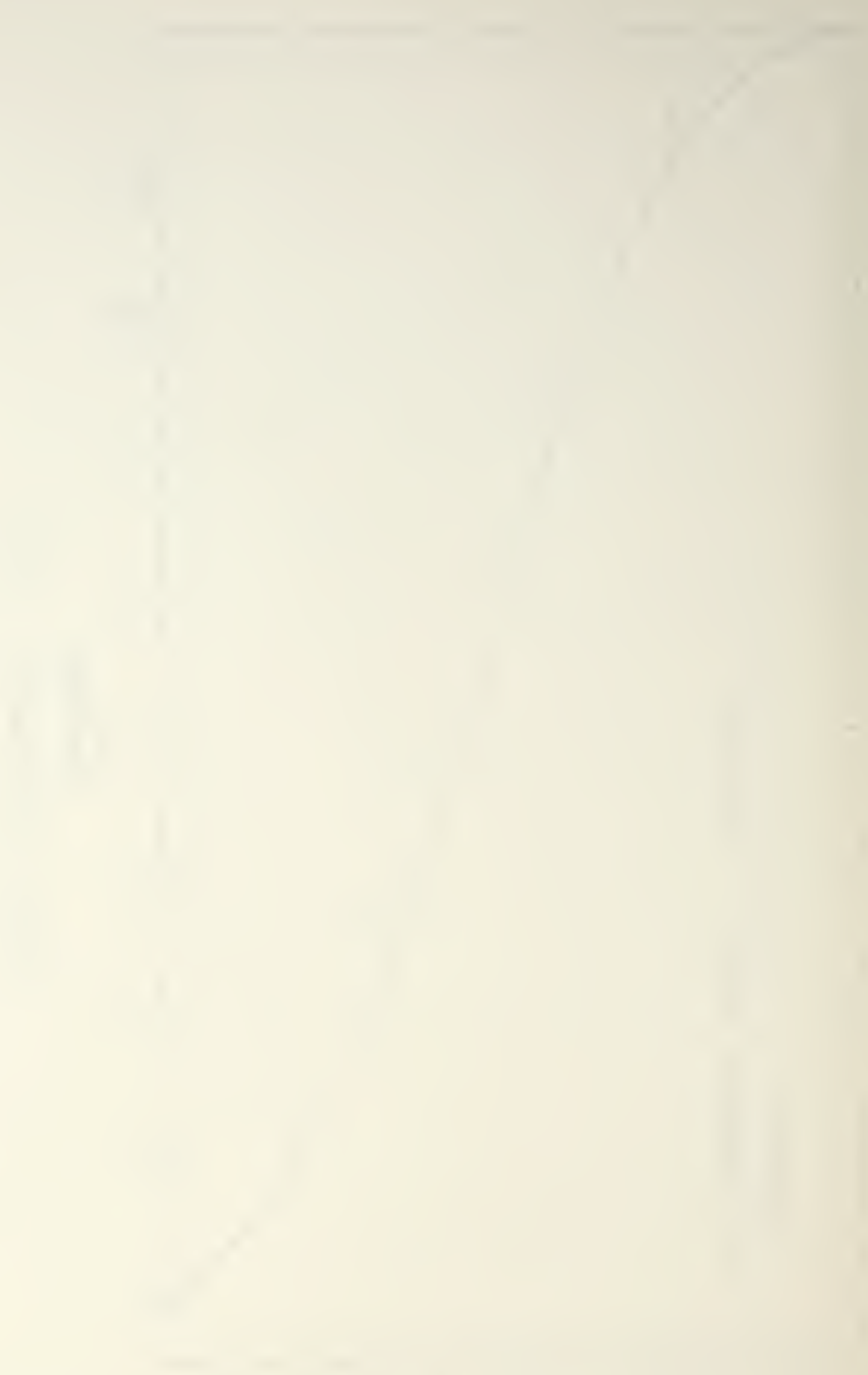


figure 10

Vortex Distribution, $\ell = 1.5$



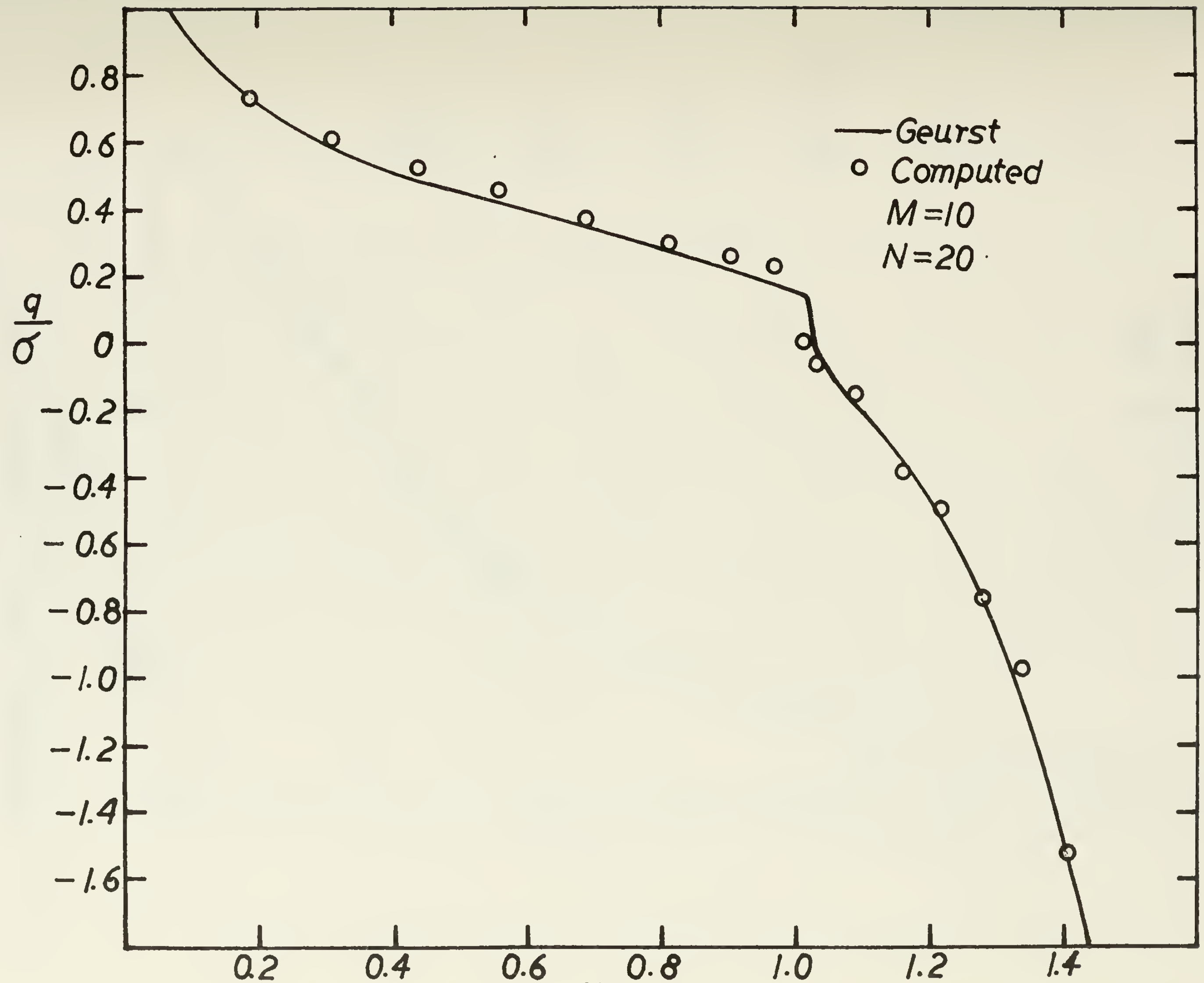


figure 11 Source Distribution, $\ell=1.5$

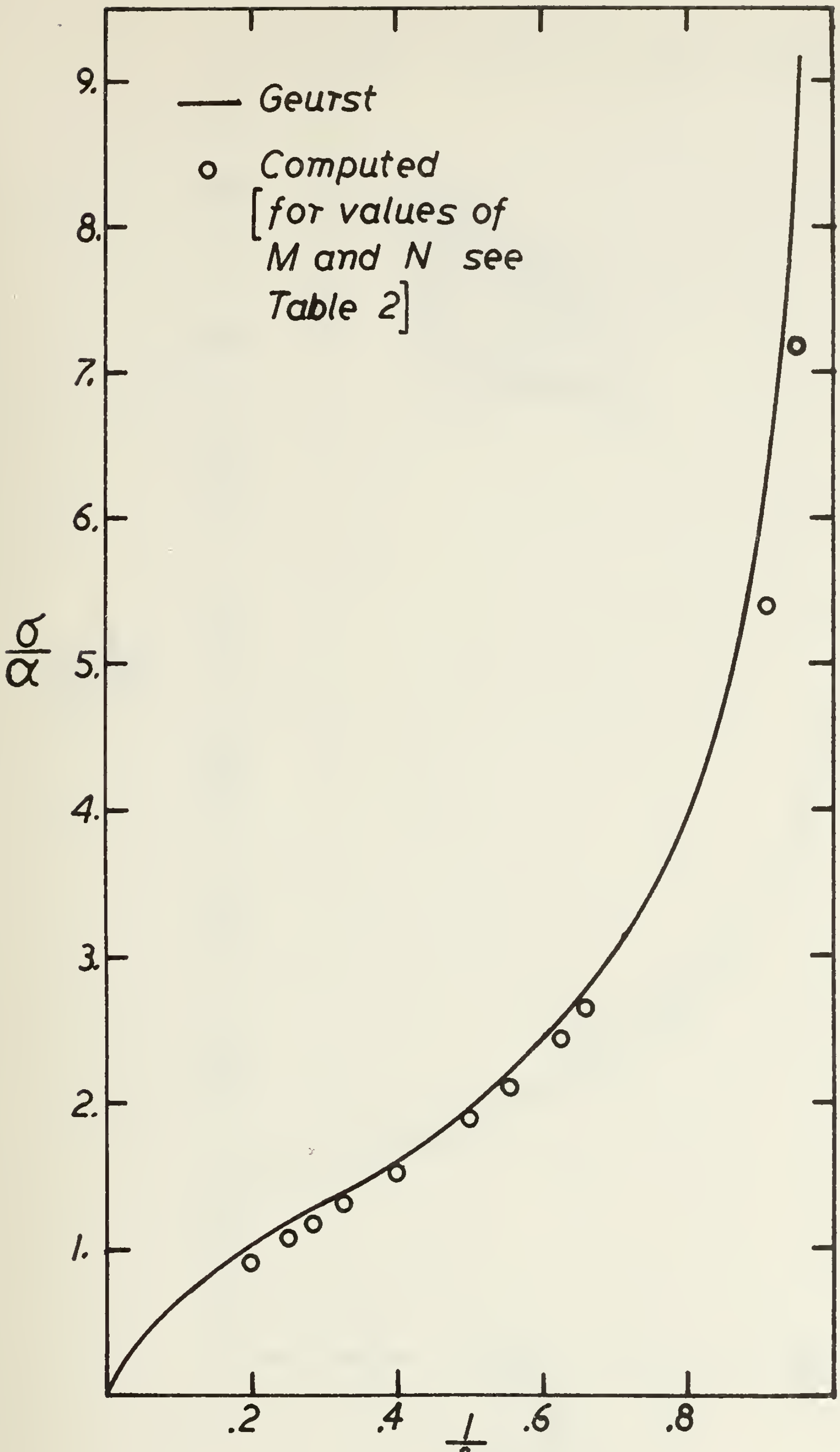


figure 12
Cavity Length, Super Cavitation

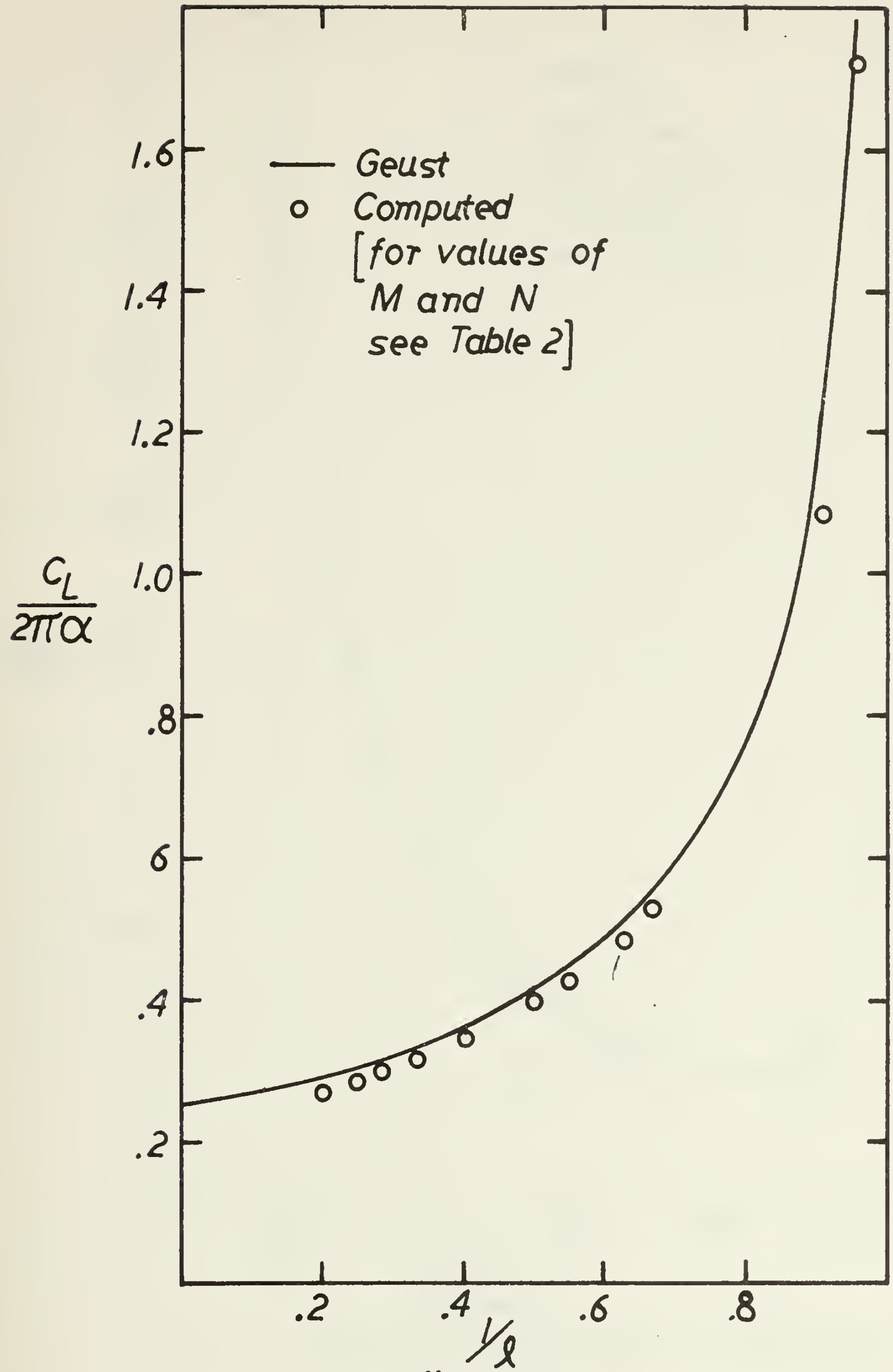


figure 13

Lift Coefficient–Super Cavitation

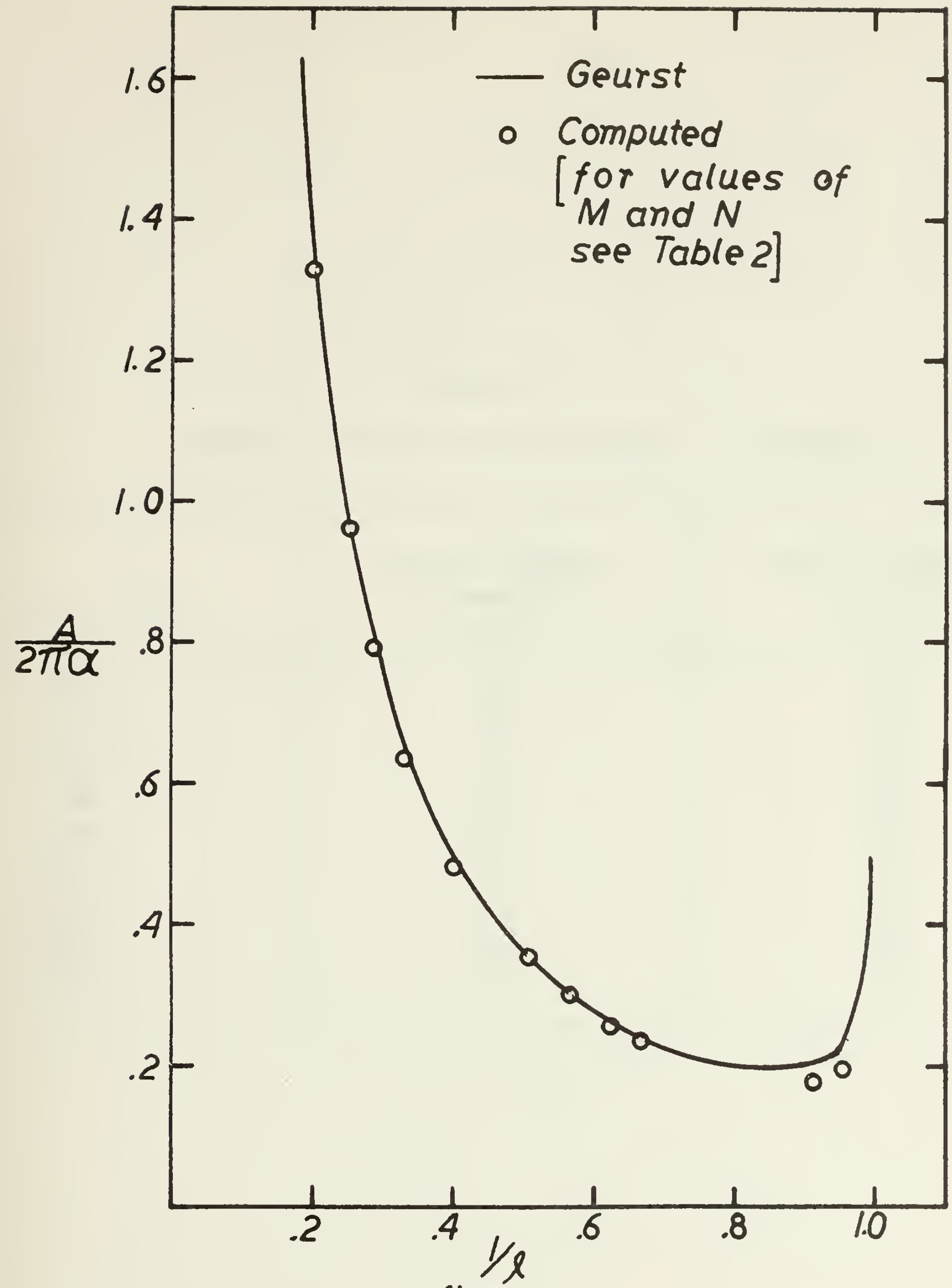


figure 14

Cavity Area–Super Cavitation

Table 2

Values of M and N for figures 10 through 14

Cavity length ℓ	No. of Vortex elements M	No. of Source elements N
1.05	20	25
1.10	20	25
1.50	15	30
1.60	15	30
1.80	15	30
2.00	15	30
2.50	15	30
3.00	15	30
3.50	15	30
4.00	15	30
5.00	15	30

[illegible]

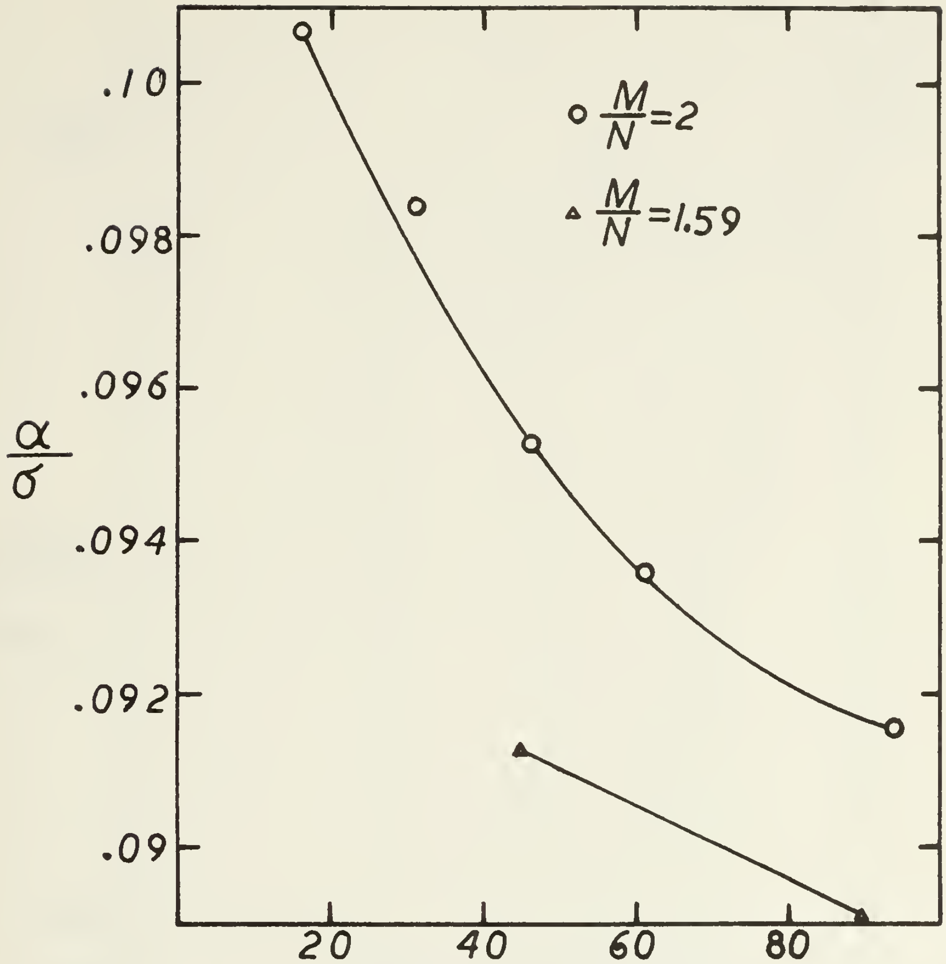


figure 15

Numerical Convergence, $\ell = 0.5$

$[(\alpha/\sigma)_{theoretical} = 0.0858]$

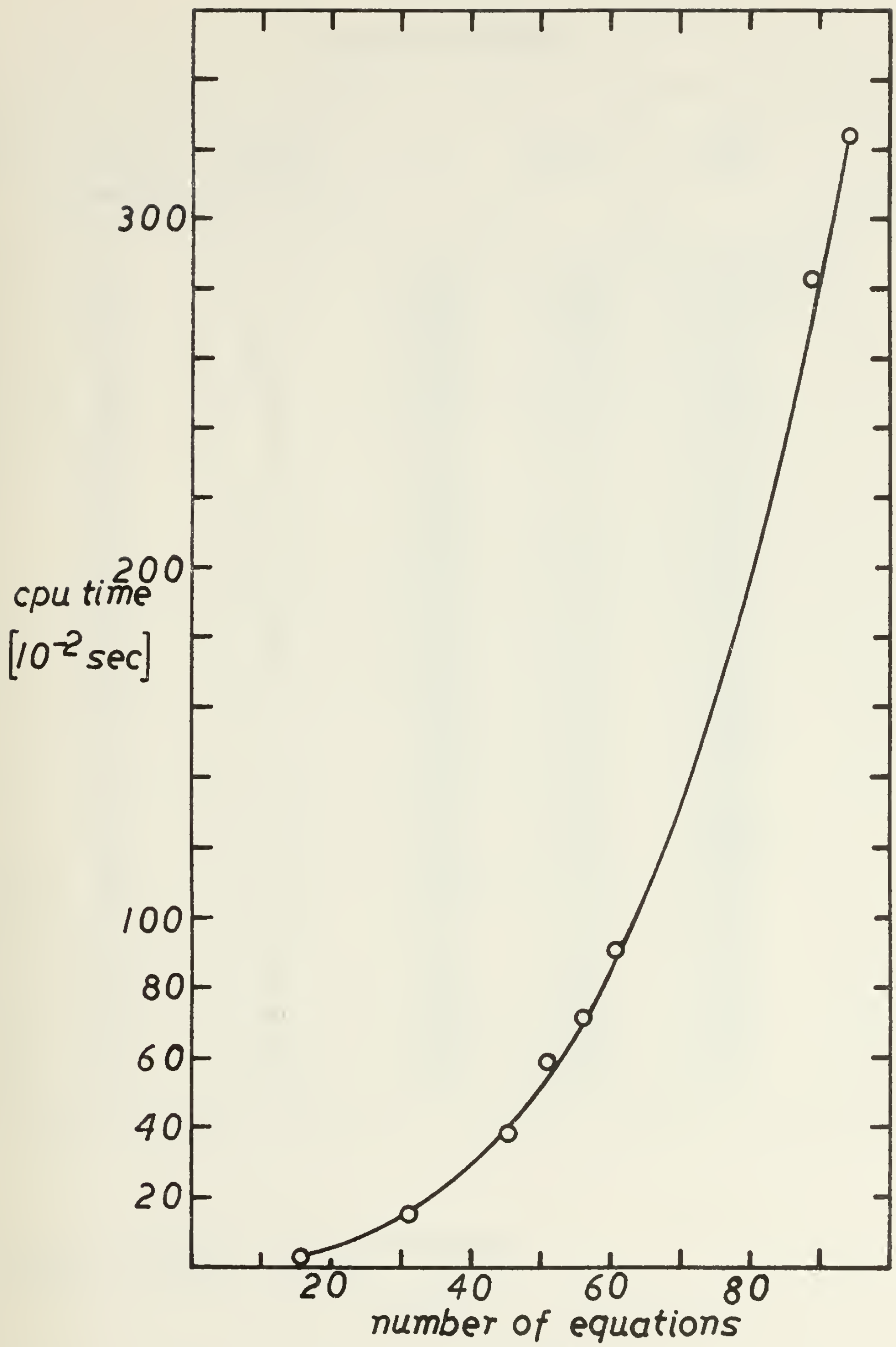


figure 16
Computation time

Table 3

Computed Results

cavity length	No. of Vortex elements M	No. of source elements N	$\frac{\alpha}{\sigma}$	$\frac{C_L}{2\pi\alpha}$	$\frac{Area}{2\pi\alpha}$	elapsed time ($\frac{1}{100}$ sec)
.1*	30	15	.0380	1.039	$4.66\text{x}10^{-3}$	40
.1	45	5	.0485	1.002	$3.45\text{x}10^{-3}$	57
.2*	30	15	.0569	1.066	.0129	41
.2*	40	10	.0627	1.035	.0119	61
.3	30	15	.0718	1.0913	.0237	42
.4*	30	15	.0844	1.12	.0368	40
.5	10	5	.107	1.09	.0412	2
.5	20	10	.0984	1.14	.0495	15
.5	30	15	.0953	1.15	.0520	42
.5	40	20	.0936	1.16	.0533	94
.5	62	31	.0916	1.18	.0549	324
.5	30	25	.0796	1.33	.0590	72
.5*	27	17	.0913	1.19	.0536	38
.5	54	34	.0891	1.20	.0558	283
.5	25	20	.0819	1.30	.0576	42
.6	30	25	.0882	1.39	.0785	70
.6*	30	15	.1047	1.20	.0695	43
.6	25	20	.0909	1.36	.0765	40
.7	30	25	.0952	1.48	.101	70
.7*	30	15	.112	1.26	.0898	42
.7	25	20	.0982	1.44	.0984	40
.75*	30	25	.0979	1.537	.114	68
.8*	30	25	.0996	1.61	.129	73
.8*	25	20	.103	1.56	.125	43
.9	30	25	.0981	1.88	.167	73
.9*	25	20	.102	1.81	.161	41
.95	30	25	.0906	2.22	.199	71
.95	25	20	.0947	2.12	.191	40

* Points plotted in the figures.

Table 3 continued

cavity length	No. of Vortex elements M	No. of source elements N	$\frac{\alpha}{\sigma}$	$\frac{C_L}{2\pi\alpha}$	$\frac{Area}{2\pi\alpha}$	elapsed time ($\frac{1}{100}$ sec)
1.05*	20	25	.136	1.55	.192	41
1.1*	20	25	.185	1.082	.177	44
1.5	10	20	.384	0.506	.230	15
1.5	5	10	.413	0.445	.217	3
1.5*	15	30	.379	0.515	.233	39
1.5*	20	40	.372	0.527	.236	89
1.5	17	27	.379	0.514	.234	38
1.5*	20	25	.405	0.461	.226	40
1.6*	15	30	.414	0.475	.254	42
1.8*	15	30	.477	0.422	.300	41
2.0*	15	30	.532	0.388	.348	43
2.5*	15	30	.653	0.338	.482	41
3.0*	15	30	.757	0.311	.620	40
3.5*	15	30	.851	0.293	.790	41
4.0*	15	30	.937	0.281	.960	40
5.0	15	30	1.093	0.263	1.330	42

Table 4

Singular Behavior

M	N	n* leading edge Vortex	n* cavity termination Vortex	n* leading edge source	n* cavity termination source
20	10	.093	31.1	-.794	36
27	17	.099	27.5	-.716	35.5
60	30	-.106	-60.5	-.604	57.7

* Based on only the two elements closest to the singularity (e.g. for the leading edge vortex distribution the elements are the first to vortex densities) and $\ell = 0.5$.

[illegible]

[illegible]

[illegible]

[illegible]

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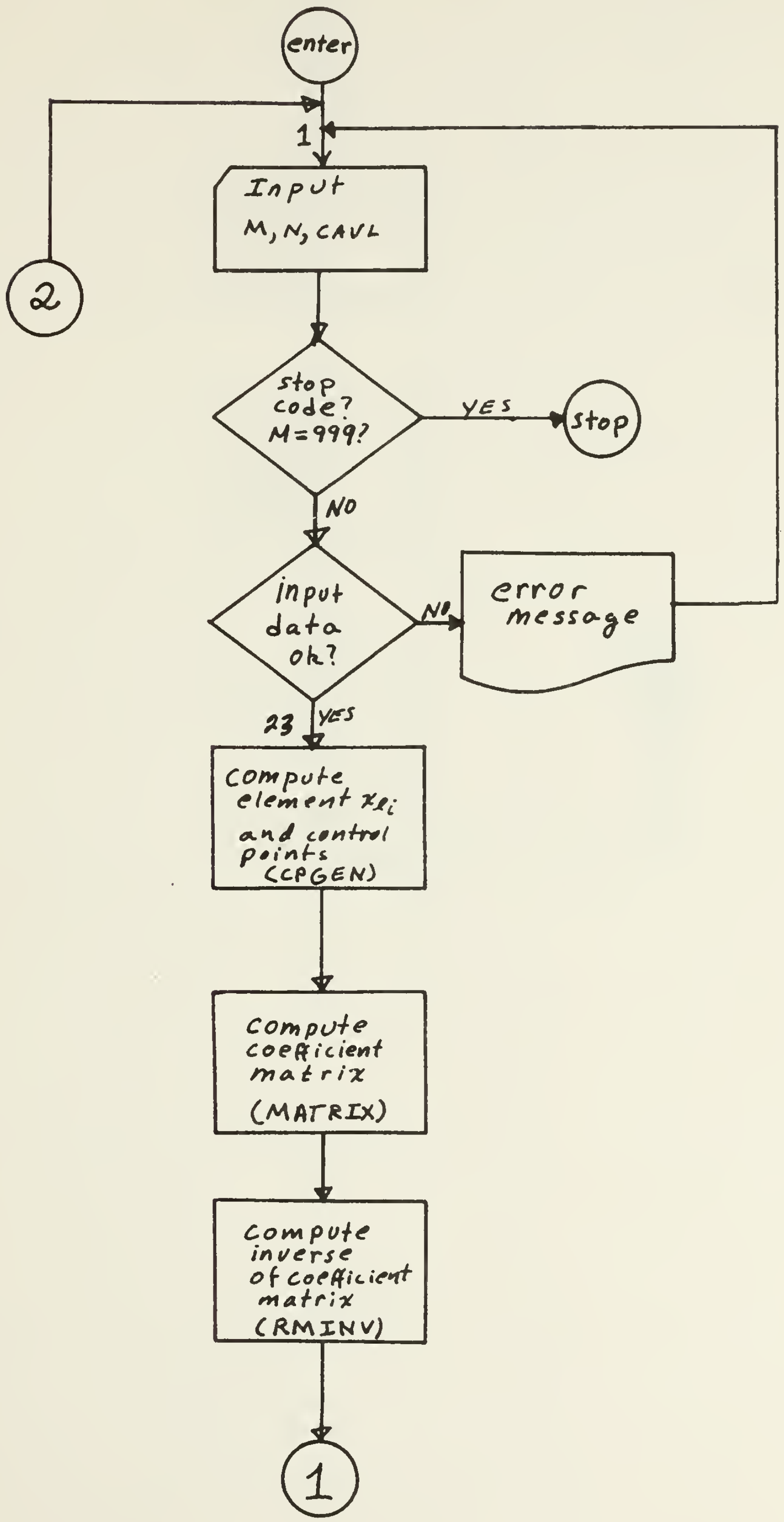
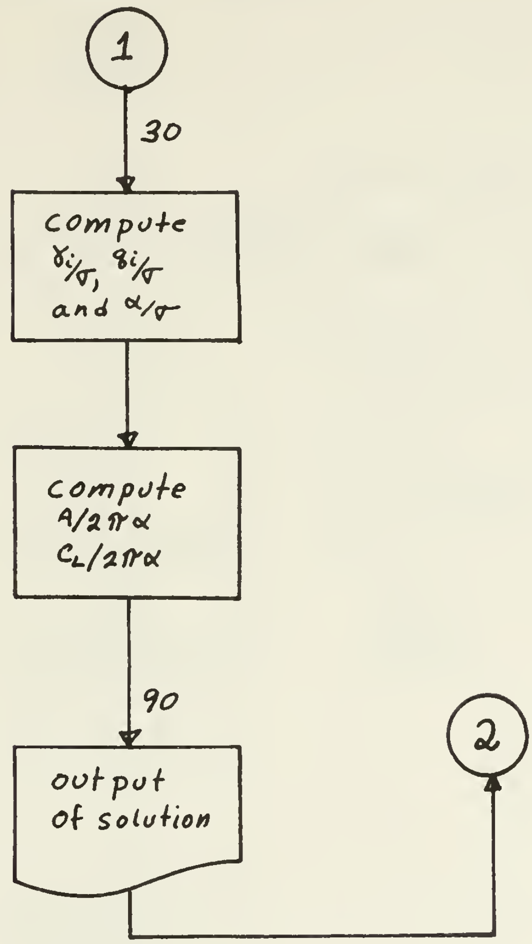
APPENDIX A
COMPUTER PROGRAM

Appendix A includes the flow charts for the computer program and the computer program listing. The flow charts are provided to give a broad overview of the program flow. The numbers between blocks in the flow charts indicate approximate statement numbers in the program listing. A flow chart for RMINV is not provided since it is a standard matrix inversion routine.

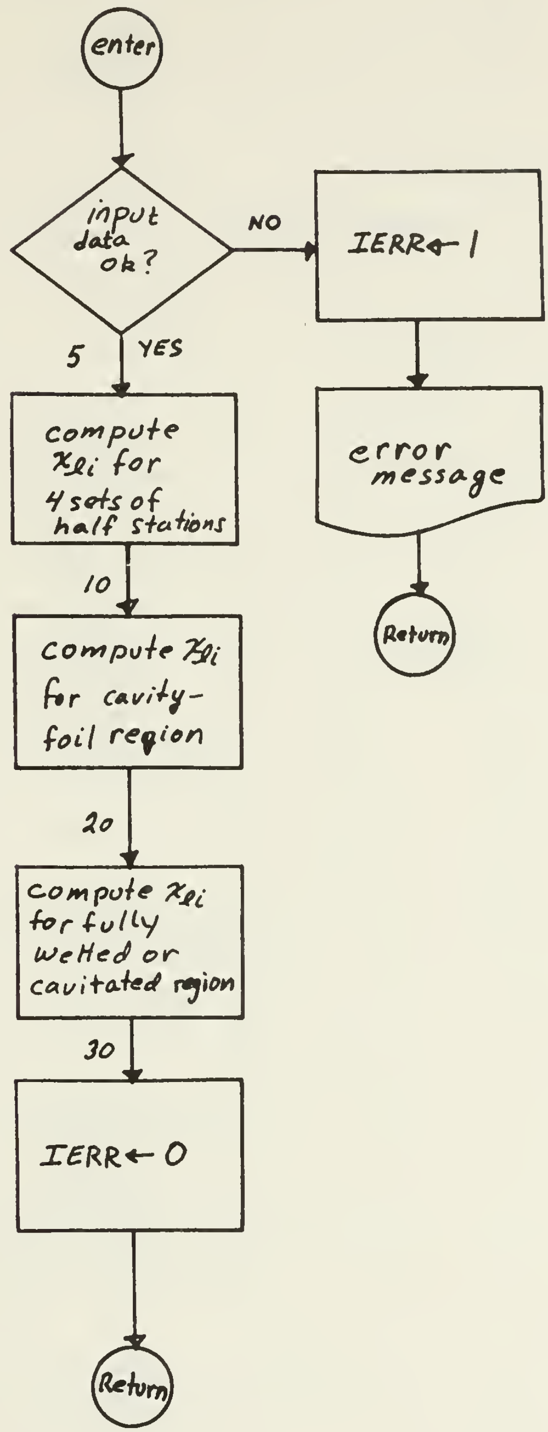
FLOW CHART, MAIN PROGRAM



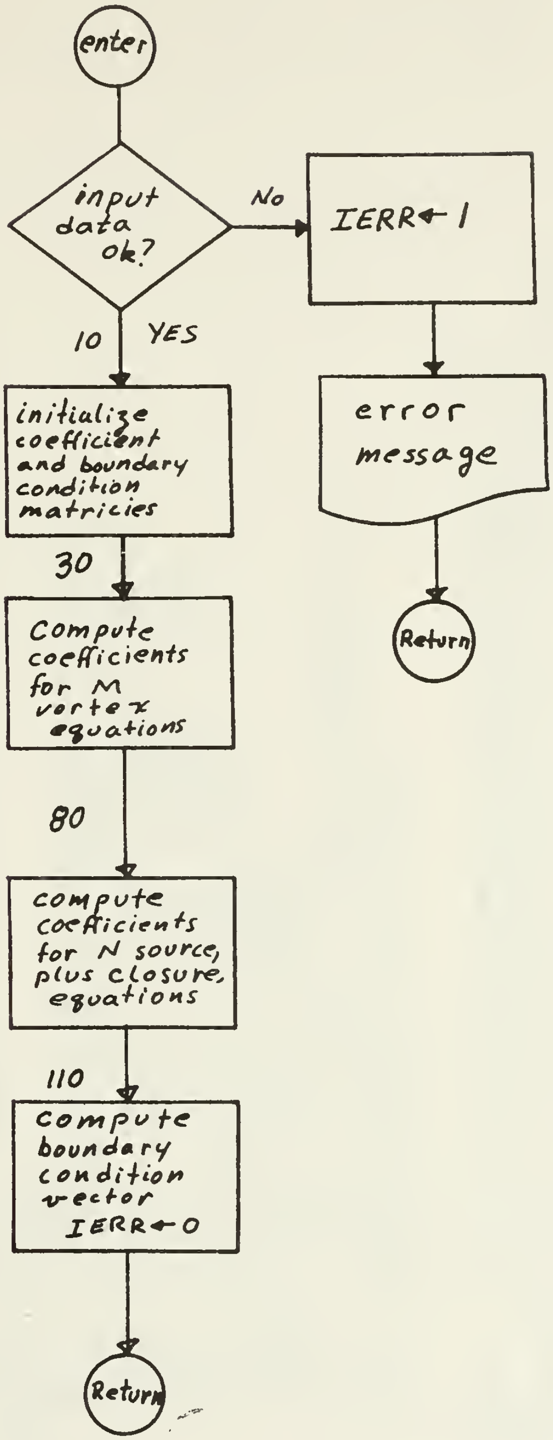
MAIN, CONTINUED



FLOW CHART, CPGEN



FLOW CHART, MATRIX



```
C     PROGRAM CAVITATICN
C     PROGRAM COMPUTES ALFA/SIGMA, LIFT COEFFICIENT/(2*PI*ALFA),
C     AREA OF CAVITY/(2*PI*ALFA)AND SOURCE AND VORTEX DISTRIBUTIONS
C     FOR A GIVEN CAVITY LENGTH AND A FLAT PLATE
C     INPUT VARIABLES
C        M=NO. OF VORTEX ELEMENTS, MUXST BE GREATER THAN 8 FOR A
C        PARTIAL CAVITY AND GREATER THAN 4 FOR A SUPER CAVITY
C        N= NO. OF SOURCE ELEMENTS, MUST BE GREATER THAN 4 FOR A
C        PARTIAL CAVITY AND GREATER THAN 8 FOR A SUPER CAVITY
C        IWRT=0 FOR NO OUTPUT OF SOURCE AND VORTEX ELEMENT COORDINATES
C        AND NO OUTPUT OF SOURCE AND VORTEX DISTRIBUTIONS
C        IWRT .NE. 0 FOR OUTPUT OF SOURCE AND VORTEX DISTRIBUTIONS
C        CAVL= CAVITY LENGTH
C     OUTPUT VARIABLES
C        XL(I) = LOWER BOUNDARY OF SOURCE-VORTEX ELEMENTS
C        X(I)= LOCATIONS OF CONTROL POINTS
C        XU(I)= UPPER BOUNDARY OF SOURCE-VORTEX ELEMENTS
C        G(I)= VORTEX DENSITY FOR ITH ELEMENT / SIGMA
C        Q(I)= SOURCE DENSITY FOR ITH ELEMENT / SIGMA
C        CL= LIFT COEFFICIENT / (2*PI*ALFA)
C        AREA=CAVITY AREA/(2*PI*ALFA)
C        SOURCE= TOTAL SOURCE STRENGTH
C        ALFA=1. ANGLE OF ATTACK IN ABOVE STATEMENTS
C        OR   2. ANGLE OF ATTACK / SIGMA IN OUTPUT LIST
C        CAVL= INPUT VALUE OF CAVITY LENGTH
C     SIGMA= CAVITATION NUMBER
C
      INTEGER   IN,OUT,ND,NDIM,MN,MN1,JJ,IWORK,IERR,IWRT
      REAL   X,BC,C,ALFA,CL,AREA,SOURCE,G,Q,GAMMA,CAVL,PI,XL,XU
      DIMENSION IWORK(101,2),G(100),Q(100),X(100),BC(101),C(101,101),
     1XL(100),XU(100),XS(100)
      DATA ND,NDIM/100,101/
      DATA IN,OUT,IPCH/5,6,7/
      PI=2.*ARCOS(0.0)
    1 READ(IN,1000) M,N,IWRT,CAVL
      IF(M .EQ. 999) STOP
```

```
      CALL TIMING (IST)
    IF(M+N .LE. 100) GO TO 10
    WRITE(OUT,2000) M,N
    GO TO 1
 10 IF (CAVL .GT. 0.0) GO TO 20
    WRITE(OUT,2010) CAVL
    GO TO 1
 20 IF(CAVL .LT. 1. .AND. N .LT. M) GO TO 22
    IF(CAVL .GT. 1. .AND. N .GT. M) GO TO 22
    WRITE(OUT,2060)
    GO TO 1
 22 MN=M+N
    MN1=MN+1
 23 WRITE(OUT,6000)
    WRITE(OUT,2020) M,N,CAVL
    CALL CPGEN(M,N,MN,ND,CAVL,X,XL,XU,XS,     IERR)
    IF(IERR .NE. 0) GO TO 1
    L=MAXO(M,N)
    IF(IWRT .NE. 0) WRITE(OUT,2030) (XL(I),X(I),XU(I),XS(I),
   1 I=1,L)
    CALL MATRIX (M,N,MN,MN1,IERR,ND,NDIM,XS, X,BC,C,
   1XL,XU,PI  )
    IF(IERR .NE. 0) GO TO 1
    CALL RMINV(NDIM,MN1,C,DTERM,IWORK,JJ)
      WRITE(OUT,2050) DTERM
    IF(JJ .NE. 0) GO TO 1
C
C   OBTAIN SOLUTION FROM INVERSE MATRIX AND BOUNDARY CONDITIONS
C
    DO 30 I=1,M
    G(I)=0.0
 30 CONTINUE
    GAMMA=0.0
    SOURCE=0.0
    AREA=0.0
    DO 40 I=1,N
```

```
      Q(I)=0.0
   40 CONTINUE
      DO 60 I=1,M
      DO 50 J=1,MN1
      G(I)=G(I)+C(I,J)*BC(J)
   50 CONTINUE
      GAMMA=GAMMA+G(I)*(XU(I)-XL(I))
   60 CONTINUE
      GAMMA=GAMMA-G(M)*(XU(M)-XL(M))/2.
      DO 80 I=1,N
      DO 70 J=1,MN1
      Q(I)=Q(I)+C(I+M,J)*BC(J)
   70 CONTINUE
      SOURCE=SOURCE+Q(I)*(XU(I)-XL(I))
      AREA=AREA+Q(I)*(XU(I)**2-XL(I)**2)
   80 CONTINUE
      ALFA=0.0
      DO 90 J=1,MN1
      ALFA=ALFA+C(MN1,J)*BC(J)
   90 CONTINUE
      CL=-GAMMA/(PI*ALFA)
      AREA=-AREA/(4.*PI*ALFA)
      IF(IWRT .NE. 0) WRITE(OUT,3000)  (G(I), I=1,M)
      IF(IWRT .NE. 0) WRITE(OUT,3010)  (Q(I), I=1,N)
      WRITE(OUT,3020) CL,AREA,SOURCE,ALFA,CAVL
         CALL TIMING(ISTP)
         ITIME=ISTP-IST
      WRITE(OUT,4000) ITIME
      GO TO 1
C
C     FORMAT STATEMENTS
C
 1000 FORMAT(3I3,11X,F10.0)
 1010 FORMAT(10X,F10.0)
 2000 FORMAT(' ',10X,'*** INPUT ERROR ***  M=',I3,2X,'N=',I3/'1')
 2010 FORMAT(' ',10X,'*** INPUT ERROR ***  CAVITY LENGTH=',F10.5/'1')
```

```
2020 FORMAT(' ',10X,'NO. OF VORTICIES=',I3/
   1        ' ',10X,'NO. OF SOURCES=  ',I3/
   2        ' ',10X,'CAVITY LENGTH=   ',F10.5/)
2030 FORMAT (' ',20X,'XL(I)', 9X,'X(I)', 9X,'XU(I)', 9X,'XS(I)'/
   1        (' ',10X,4E15.5))
2050 FORMAT('0',10X,'*** DETERMINANT=',E13.7,'***'/ )
2060 FORMAT(' ',10X,'*** INPUT ERRCR M, N, CAVL ***'/'1')
3000 FORMAT('0',10X,'VORTEX DENSITIES/SIGMA:'/
   1       (' ',10X,4E15.5))
3010 FORMAT('0',10X,'SOURCE DENSITIES/SIGMA:'/
   1       (' ',10X,4E15.5))
3020 FORMAT(' '/
   1        ' ',15X,'LIFT COEFICIENT/(2*PI*ALFA) =',E13.7//
   2        ' ',15X,'AREA OF CAVITY/(2*PI*ALFA)  =',E13.7//
   3        ' ',15X,'TOTAL SOURCE STRENGTH/SIGMA =',E13.7//
   4        ' ',15X,'ANGLE OF ATTACK/SIGMA       =',E13.7//
   5        ' ',15X,'CAVITY LENGTH               =',E13.7/)
4000 FORMAT(' ',10X,'ELAPSED TIME= ',I5,2X,'1/100 SEC.'/'1')
6000 FORMAT('0',/)
     END
```

```
C     SUBROUTINE CPGEN
C     SUBPROGRAM PLACES CONTROL POINTS ON FOIL AND SELECTS
C     ELEMENT SIZE FOR EACH VORTEX-SOURCE ELEMENT
C     INPUT VARIABLES
C        M= NO. OF VORTICIES
C        N= NO OF SOURCES
C        MN= M+N
C        K= DIMENSION OF CONTROL POINT VECTOR
C        CAVL= CAVITY LENGTH
C     VARIABLES RETURNED
C        XL(I)= LOWER BOUNDARY OF SOURCE-VORTEX ELEMENTS
C        X(I)= CONTRCL POINT VECTOR
C        XU(I)= UPPER BOUNDARY OF SOURCE VORTEX ELEMENTS
C        XS(I)= SOURCE CONTROL POINTS
C        IERR= ERROR CODE
C        IERR=0 NO ERRORS INDICATED
C        IERR=1 ERROR IN DIMENSION OR INPUT OF M OR N
C
      SUBROUTINE CPGEN(M,N,MN,K,CAVL,X,XL,XU,XS,    IERR)
      DIMENSION X(K),XL(K),XU(K),XS(K)
      IF(MN .LE. K) GO TO 5
      IERR=1
      WRITE(6,2000)
      RETURN
    5 L=MINO(M,N)
      LL=MN-L
      Z=AMIN1(1.,CAVL)
      LL1=LL+1
      L1=L+1
      Z1=1.+CAVL-Z
      F=L-2
      F1=Z/(2.*F)
      F=LL-L-2
      F2=(Z1-Z)/(2.*F)
      XL(1)=0.0
      XL(L1)=Z
```

```
      DO 10 I=1,2
      E=I
      XL(I+1)=E*F1
      XL(L1-I)=Z-E*F1
      XL(L1+I)=Z+E*F2
      XL(LL1-I)=Z1-E*F2
   10 CONTINUE
      ZONE=XL(L-1)-XL(3)
      E4=L-4
      L2=L-2
      DO 20 I=4,L2
      E=I-3
      XL(I)=XL(3)+E*ZONE/E4
   20 CONTINUE
      ZONE=XL(LL-1)-XL(L+3)
      E4=LL-L-4
      L3=L+3
      L4=L+4
      LL2=LL-2
      DO 30 I=L4,LL2
      E=I-L3
      XL(I)=XL(L3)+E*ZONE/E4
   30 CONTINUE
      LL1=LL-1
      DO 40 I=1,LL1
      XU(I)=XL(I+1)
      X(I)=XL(I)+(XU(I)-XL(I))*0.9
      XS(I)=XL(I)+(XU(I)-XL(I))*0.5
   40 CONTINUE
      XU(LL)=Z1
      X(LL)=XL(LL)+(Z1-XL(LL))*0.9
      XS(LL)=(Z1+XL(LL))/2.
      XS(1)=XU(1)*0.9
      XS(N)=XL(N)+(XU(N)-XL(N))*0.1
      IERR=0
      RETURN
```

```
2000 FORMAT('0',10X,'*** CPGEN DIMENSION ERROR ***')
     END
```

```
C      SUBROUTINE MATRIX
C      SUBPROGRAM TO COMPUTE COEFFICIENT MATRIX
C      INPUT VARIABLES
C         M=NO. OF VORTICIES
C         N=NO. OF SOURCES
C         MN=M+N
C         MN1=M+N+1
C         K=DIMENSION OF CONTROL POINT VECTOR
C          NDIM= DIMENSION OF COEFFICIENT MATRIX AND BOUNDARY CONDITION
C         VECTOR
C         XL(I)= LOWAR BOUNDARY OF SOURCE-VORTEX ELEMENTS
C         X(I)=CONTROL POINT VECTOR
C         XU(I)= UPPER BOUNDARY OF SOURCE-VORTEX ELEMENTS
C         XS(I)= SOURCE CONTROL POINTS
C         PI
C      RETURNED VARIABLES
C         C(I,J)=COEFFICIENT MATRIX
C         B(J)= BOUNDARY CONDITION VECTOR
C
      SUBROUTINE MATRIX(M,N,MN,MN1,IERR,K,NDIM,XS, X,BC,C,XL,
     1XU,PI  )
      DIMENSION C(NDIM,NDIM),BC(NDIM),X(K),XL(K),XU(K),XS(K)
      COEFF(A,B,C)=ALOG((A-B)/(A-C))
      IF(MN1 .LE. NDIM .AND. MN .LE. K) GO TO 10
      IERR=1
      WRITE(6,2000)
      RETURN
   10 IERR=0
      CII=ALOG( 9.0)
      DO 30 I=1,MN1
      BC(I)=0.0
      DO 20 J=1,MN1
      C(I,J)=0.0
   20 CONTINUE
   30 CONTINUE
C
```

```
C     COMPUTE COEFFICIENTS FOR VORTEX EQUATIONS
C
      L=M-1
      DO 50 J=1,L
      DO 40 I=1,M
      IF(I .NE. J) C(J,I)=COEFF(X(J),XL(I),XU(I))
      IF(I .EQ. J) C(I,I)=CII
      IF(I .EQ. M) C(J,I)=(C(J,I)*(1.-X(J))      )/(XU(M)-XL(M))+1.
   40 CONTINUE
   50 CONTINUE
      DO 60 I=1,L
      C(M,I)=COEFF(X(M),XL(I),XU(I))
   60 CONTINUE
      C(M,M)=1.+(1.-X(M))*CII/(XU(M)-XL(M))
C
C     PUT IN THE EFFECT OF LOCAL SOURCES
C
      L=M+1
      DO 70 I=L,MN
      C(I-M,I)=-PI
   70 CONTINUE
C
C     PUT IN COEFFICIENT OF ALFA/SIGMA
C
      DO 80 I=1,M
      C(I,MN1)=2.*PI
   80 CONTINUE
C
C     VORTEX EQUATIONS ARE COMPLETE
C     COMPUTE COEFFICIENTS FOR SOURCE EQUATIONS
C
      J1=0
      DO 100 J=L,MN
      J1=J1+1
      I1=0
      DO 90 I=L,MN
```

```
      I1=I1+1
      IF(J .NE. I) C(J,I)=COEFF(XS(J1),XL(I1),XU(I1))
   90 CONTINUE
      IF(J-M .LT. M) C(J,J-M)=-PI
  100 CONTINUE
      C(L,L)=CII
      C(MN,MN)=-CII
      IF(N .GT. M) C(M+M,M)=-PI/2.
C
C     FILL IN LAST SOURCE EQUATION (CLOSURE)
C
      I=0
      DO 110 J=L,MN
      I=I+1
      C(MN1,J)=XU(I)-XL(I)
  110 CONTINUE
C
C     FILL IN BOUNDARY CONDITIONS
C
      DO 120 J=L,MN
      BC(J)=PI
  120 CONTINUE
      RETURN
 2000 FORMAT('0',10X,'*** MATRIX DIMENSION ERROR ***')
      END
```

```
      SUBROUTINE RMINV(NDIM, NORDER, A, DETERM, IWORK, IERR)
C
      REAL A, BIGA, DETERM, ABS, HOLD
      INTEGER I, J, K, NDIM, NORDER, KK, NM1
C
      DIMENSION A(NDIM, NDIM),   IWORK(NDIM, 2)
C
C   CHECK FOR ARGUMENT ERRORS.
      IF (NDIM .GE. NORDER .AND. NORDER .GT. 0) GO TO 10
      IERR = 1
      WRITE (6, 1001) NDIM, NORDER
      RETURN
   10 IF (NORDER.NE.1) GO TO 11
      DETERM=A(1,1)
      A(1,1)=1.0/A(1,1)
      IERR=0
      RETURN
   11 DETERM = 1.0E0
C
      DO 80 K = 1, NORDER
C
C   SEARCH FOR LARGEST ELEMENT.
         BIGA = - 1.0E0
         DO 25 J = K, NORDER
            DO 20 I = K, NORDER
               HOLD = ABS(A(I, J))
               IF (BIGA .GE. HOLD) GO TO 20
               BIGA = HOLD
               IWORK(K, 1) = I
               IWORK(K, 2) = J
   20       CONTINUE
   25    CONTINUE
C
C   INTERCHANGE ROWS.
         I = IWORK(K, 1)
         IF (I .LE. K) GO TO 35
```

```
         DO 30 J = 1, NORDER
            HOLD = - A(K, J)
            A(K, J) = A(I, J)
            A(I, J) = HOLD
   30    CONTINUE
C
C  INTERCHANGE COLUMNS.
   35    J = IWORK(K, 2)
         IF (J .LE. K) GO TO 45
         DO 40 I = 1, NORDER
            HOLD = - A(I, K)
            A(I, K) = A(I, J)
            A(I, J) = HOLD
   40    CONTINUE
C
C  FETCH PIVOT.
   45    BIGA = A(K, K)
C
C  DETERMINANT IS PRODUCT OF PIVOTS.
         DETERM = DETERM * BIGA
C
C  IF DETERMINANT VANISHES, MATRIX IS SINGULAR.
         IF (DETERM .NE. 0.0E0) GO TO 50
         IERR = 2
         WRITE (6, 1002)
         RETURN
C
C  REPLACE PIVOT BY RECIPROCAL.
   50    A(K, K) = 1.0E0 / BIGA
C
C  DIVIDE COLUMN BY MINUS PIVOT.
         DO 55 I = 1, NORDER
            IF (I .EQ. K) GO TO 55
            HOLD = - A(I, K) / BIGA
            A(I, K) = HOLD
C
```

```
C   REDUCE MATRIX.
            DO 65 J = 1, NORDER
               IF (J .NE. K) A(I, J) = A(I, J) + HOLD * A(K, J)
   65       CONTINUE
   55    CONTINUE
C
C   DIVIDE ROW BY PIVOT.
         DO 75 J = 1, NORDER
            IF (J .NE. K) A(K, J) = A(K, J) / PIGA
   75    CONTINUE
   80 CONTINUE
C
C   FINAL ROW AND COLUMN INTERCHANGE.
      NM1 = NORDER - 1
      DO 140 KK = 1, NM1
         K = NORDER - KK
C
         J = IWORK(K, 1)
         IF (J .LE. K) GO TO 120
         DO 110 I = 1, NORDER
            HOLD = A(I, K)
            A(I, K) = - A(I, J)
            A(I, J) = HOLD
  110    CONTINUE
C
  120    I = IWORK(K, 2)
         IF (I .LE. K) GO TO 140
         DO 130 J = 1, NORDER
            HOLD = A(K, J)
            A(K, J) = - A(I, J)
            A(I, J) = HOLD
  130    CONTINUE
  140 CONTINUE
C
      IERR = 0
      RETURN
```

```
C                                                                       RMNV1090
 1001 FORMAT(23H RMINV:  ARGUMENT ERROR, 2I11)                          RMNV1100
 1002 FORMAT(28H RMINV:  MATRIX IS SINGULAR.)                           RMNV1110
C                                                                       RMNV1120
      END                                                               RMNV1130
```

APPENDIX B

SAMPLE COMPUTER OUTPUT

Appendix B contains the sample computer output. This output contains the results of a convergence test for cavity lengths equal to ½ chord length and 1.5 times chord length. Note that for a partial cavity source control points beyond cavity termination are listed in the output. These control points are superfluous and not used in the program. The same is true of control points for the vortex elements beyond chord length in super cavitation.

The listing of source and vortex densities has the same order as the listing of control points.

```
NO. CF VORTICIES= 10
NO. OF SOURCES=    5
CAVITY LENGTH=      0.50000

*** DETERMINANT=-.1317091E+09***

     LIFT COEFICIENT/(2*PI*ALFA) =0.1092649E+01

     AREA OF CAVITY/(2*PI*ALFA)  =C.4117717E-01

     TOTAL SOURCE STRENGTH/SIGMA =C.2272427E-06

     ANGLE OF ATTACK/SIGMA       =0.1067560E+00

     CAVITY LENGTH               =0.5000000E+00

ELAPSEC TIME=     2  1/100 SEC.
```

```
NO. OF VORTICIES= 20
NO. CF SOURCES=   1C
CAVITY LENGTH=     0.50000

       XL(I)           X(I)           XU(I)          XS(I)
   0.0            0.28125E-01    0.31250E-01    0.28125E-01
   0.31250E-01    0.59375E-01    0.62500E-01    0.46875E-01
   0.62500E-01    0.11875E+00    C.12500E+00    0.93750E-01
   0.12500E+00    0.18125E+00    0.18750E+00    0.15625E+00
   0.18750E+00    0.24375E+00    C.25000E+00    0.21875E+00
   C.25000E+00    0.30625E+00    0.31250E+00    0.28125E+00
   0.31250E+00    0.36875E+00    C.37500E+00    0.34375E+00
   0.37500E+00    0.43125E+00    0.43750E+00    0.40625E+00
   0.43750E+00    0.46562E+00    0.46875E+00    0.45313E+00
   0.46875E+00    0.49687E+00    C.50000E+00    0.47187E+00
   0.50000E+00    0.52812E+00    0.53125E+00    0.51563E+00
   0.53125E+00    0.55937E+00    C.56250E+00    0.54688E+00
   0.56250E+00    0.61875E+00    C.62500E+00    0.59375E+00
   0.62500E+00    0.68125E+00    0.68750E+00    0.65625E+00
   0.68750E+00    0.74375E+00    C.75000E+00    0.71875E+00
   0.75000E+00    0.80625E+00    0.81250E+00    0.78125E+00
   0.81250E+00    0.86875E+00    0.87500E+00    0.84375E+00
   0.87500E+00    0.93125E+00    0.93750E+00    0.90625E+00
   0.93750E+00    0.96562E+00    0.96875E+00    0.95313E+00
   0.96875E+00    0.99687E+00    C.10000E+01    0.98438E+00

*** DETERMINANT=0.3532951E+17***


VORTEX DENSITIES/SIGMA:
  -0.93763E+00   -0.87444E+00   -0.79342E+00   -0.72474E+00
  -0.6856CE+00   -0.66212E+00   -C.63783E+00   -0.65261E+00
  -0.64077E+0C   -0.55656E+00    0.65990E+00    0.11008E+00
  -0.45946E-02   -0.59571E-01   -0.71848E-01   -0.70911E-01
  -0.63426E-01   -0.51625E-01   -C.37786E-01   -0.38749E-01

SOURCE DENSITIES/SIGMA:
   0.80515E+00    0.53662E+00    C.33202E+00    0.20736E+00
   0.10949E+00    0.76209E-02   -C.89198E-01   -0.30393E+00
  -0.56007E+00   -C.13084E+01

    LIFT COEFFICIENT/(2*PI*ALFA) =0.1137383E+01

    AREA CF CAVITY/(2*PI*ALFA)   =C.4946671E-01

    TOTAL SOURCE STRENGTH/SIGMA  =0.1750886E-06

    ANGLE OF ATTACK/SIGMA        =C.9841657E-01

    CAVITY LENGTH                =C.5000000E+00

ELAPSED TIME=    15  1/100  SEC.
```

```
NO. OF VORTICIES= 10
NO. OF SOURCES=   20
CAVITY LENGTH=     1.50000

      XL(I)          X(I)           XU(I)          XS(I)
   0.0            0.56250E-01    0.62500E-01    0.56250E-01
   0.62500E-01    0.11875E+00    0.12500E+00    0.93750E-01
   0.12500E+00    0.23750E+00    0.25000E+00    0.18750E+00
   0.25000E+00    0.36250E+00    0.37500E+00    0.31250E+00
   0.37500E+00    0.48750E+00    0.50000E+00    0.43750E+00
   0.50000E+00    0.61250E+00    0.62500E+00    0.56250E+00
   0.62500E+00    0.73750E+00    0.75000E+00    0.68750E+00
   0.75000E+00    0.86250E+00    0.87500E+00    0.81250E+00
   0.87500E+00    0.93125E+00    0.93750E+00    0.90625E+00
   0.93750E+00    0.99375E+00    0.10000E+01    0.96875E+00
   0.10000E+01    0.10281E+01    0.10313E+01    0.10156E+01
   0.10313E+01    0.10594E+01    0.10625E+01    0.10469E+01
   0.10625E+01    0.11187E+01    0.11250E+01    0.10938E+01
   0.11250E+01    0.11812E+01    0.11875E+01    0.11563E+01
   0.11875E+01    0.12437E+01    0.12500E+01    0.12188E+01
   0.12500E+01    0.13062E+01    0.13125E+01    0.12813E+01
   0.13125E+01    0.13687E+01    0.13750E+01    0.13438E+01
   0.13750E+01    0.14312E+01    0.14375E+01    0.14063E+01
   0.14375E+01    0.14656E+01    0.14688E+01    0.14531E+01
   0.14688E+01    0.14969E+01    0.15000E+01    0.14719E+01

*** DETERMINANT=0.3851246E+13***


VORTEX DENSITIES/SIGMA:
  -0.11499E+01   -0.10381E+01   -0.87268E+00   -0.72748E+00
  -0.62812E+00   -0.54372E+00   -0.46015E+00   -0.37014E+00
  -0.26088E+00   -0.25544E+00

SOURCE DENSITIES/SIGMA:
   0.13267E+01    0.96160E+00    0.73057E+00    0.60112E+00
   0.50964E+00    0.43215E+00    0.36176E+00    0.28510E+00
   0.25084E+00    0.23408E+00    0.19550E-04   -0.33281E-01
  -0.16451E+00   -0.37803E+00   -0.49625E+00   -0.75565E+00
  -0.98532E+00   -0.15239E+01   -0.29826E+01   -0.56046E+01

     LIFT COEFICIENT/(2*PI*ALFA) =0.5062250E+00

     AREA OF CAVITY/(2*PI*ALFA)  =0.2296074E+00

     TOTAL SOURCE STRENGTH/SIGMA =-.1788139E-06

     ANGLE OF ATTACK/SIGMA       =0.3843980E+00

     CAVITY LENGTH               =0.1500000E+01

ELAPSED TIME=    15  1/100 SEC.
```

```
NO. OF VORTICIES= 15
NO. OF SOURCES=   30
CAVITY LENGTH=     1.50000


*** DETERMINANT=0.7520241E+19***

     LIFT COEFICIENT/(2*PI*ALFA) =0.5145274E+00

     AREA OF CAVITY/(2*PI*ALFA)  =C.2330343E+00

     TOTAL SOURCE STRENGTH/SIGMA =C.1430511E-05

     ANGLE OF ATTACK/SIGMA       =0.3788325E+00

     CAVITY LENGTH               =0.1500000E+01

ELAPSED TIME=    39  1/100 SEC.
```

APPENDIX C

COMPUTER PROGRAM FOR COMPUTATION OF CAVITY LENGTH FROM ARBITRARY VALUES OF ANGLE OF ATTACK AND CAVITATION NUMBER

The iterative method for computing cavity length from arbitrary values of angle of attack and cavitation number is based on assuming an initial cavity length ($\ell = 0.5$) then computing the next cavity length as:

$$(1C) \qquad \ell_{i+1} = \ell_i \left[\frac{(\frac{\alpha}{\sigma})_{input}}{(\frac{\alpha}{\sigma})_i} \right]^2$$

In equation (1 C) the value of $(\frac{\alpha}{\sigma})_i$ is the value computed from ℓ_i. This works well for moderate length partial cavities.

For super cavities the method used is to set upper and lower boundaries on the cavity length. Then the next assumed value of cavity length is:

$$\ell_{i+1} = \frac{\ell_{upper} + \ell_{lower}}{2}$$

The lower and upper boundaries on cavity length are determined by comparing $(\frac{\alpha}{\sigma})_i$ to $(\frac{\alpha}{\sigma})$ input. For $(\frac{\alpha}{\sigma})$ input greater than $(\frac{\alpha}{\sigma})_i$ the actual cavity length is greater than ℓ_i and so long as ℓ_i is greater than ℓ_{lower} the lower boundary for the cavity length is then ℓ_i . For $(\frac{\alpha}{\sigma})_{input}$ less than $(\alpha/\sigma)_i$ ℓ_i becomes the new upper boundary. Thus the boundaries on cavity length will converge to a solution.

The following program listing uses this procedure. This program also uses the subroutines CPGEN, MATRIX and RMINV.

The iterative procedure for the super cavitating case was developed to overcome apparent nonconvergence of the method used in partial cavitation. However, the resulting nonconvergence was based on theoretical cavity lengths only slightly greater than chord length (e.g. $\ell \simeq 1.04$). In this region of cavity length α/σ increases very rapidly with cavity length. Convergence of any method for these cavity lengths will be slow. Therefore, the method used herein is probably an artificial result and not a required procedure.

Table C 1

Results of Cavity Length Computations

input $\frac{\alpha}{\sigma}$	computed $\frac{\alpha}{\sigma}$	computed cavity length	No. of iterations	Geurst cavity length	Percent difference
.025	.0249	.0444	4	.045	-1.3
.05	.0498	.145	3	.155	-6.5
.075	.0745	.311	3	.375	-17.1
.10	.998	.730	3	1.04	-29.8
.15	.1498	1.06	6	1.099	-3.5
.20	.199	1.12	9	1.16	-3.4
.25	.249	1.19	6	1.25	-4.8
.333	.335	1.34	6	1.45	-7.6
.50	.498	1.875	4	2.0	-6.25
1.0	.9998	4.39	8	5.0	-12.0

[illegible]

[illegible]

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]		[illegible]	[illegible]	[illegible]

```
      INTEGER OUT
      DIMENSION IWORK(101,2),G(100),Q(100),X(100),BC(101),C(101,101),
     1XL(100),XU(100),XS(100)
      DATA ND,NDIM/100,101/
      DATA IN,OUT,IPCH/5,6,7/
      PI=2.*ARCOS(0.0)
    1 READ(IN,1000) ALFA,SIGMA
      IF (ALFA .EQ. 0.0 .OR. SIGMA .EQ.  0.0) STOP
      ICOUNT=0
      ALSIG=ALFA/SIGMA
      CAVLL=1.
      CAVLU=20.
      M=27
      N=17
      CAVL=0.5
    5 MN=M+N
      MN1=MN+1
   10 CALL CPGEN(M,N,MN,ND,CAVL,X,XL,XU,XS,     IERR)
      IF(IERR .NE. 0) GO TO 1
      CALL MATRIX (M,N,MN,MN1,IERR,ND,NDIM,XS, X,BC,C,
     1XL,XU,PI  )
      IF(IERR .NE. 0) GO TO 1
      CALL RMINV(NDIM,MN1,C,DTERM,IWORK,JJ)
      ALSIGC=0.0
      DO 20 I=1,MN1
      ALSIGC=ALSIGC+C(MN1,I)*BC(I)
   20 CONTINUE
      ICOUNT=ICOUNT+1
      IF(CAVL .GT. 1.) GO TO 25
      CAVLC=CAVL*(ALSIG/ALSIGC)**2
      IF(CAVLC .GT. 1.) GO TO 55
   25 IF(ABS(ALSIGC-ALSIG)/ALSIG .LE. 1.E-02) GO TO 100
      IF(ICOUNT .GE. 10) GO TO 90
      IF(CAVLC .LE. 0.3) GO TO 30
      IF(CAVLC .GT. 0.55) GO TO 40
      M=20
```

```
      N=10
      GO TO 80
   30 M=25
      N=10
      GO TO 80
   40 IF(CAVLC .GT. 1.0) GO TO 50
      M=25
      N=20
      GO TO 80
   50 IF(ALSIGC .LT. ALSIG) GO TO 52
      CAVLU=CAVLC
      GO TO 54
   52 CAVLL=CAVLC
   54 CAVLC=(CAVLL+CAVLU)/2.
      GO TO 56
   55 CAVLC=2.
   56 IF(CAVLC .GT. 1.5) GO TO 60
      M=20
      N=25
      GO TO 80
   60 M=15
      N=30
   80 WRITE(OUT,2030) CAVLC
      CAVL=CAVLC
      GO TO 5
   90 WRITE(OUT,2000) CAVLC,ALSIGC, ALSIG
      GO TO 1
  100 WRITE(OUT,2010)   CAVLC, ALSIGC, ALSIG
      WRITE(OUT,2020)
      GO TO 1
 1000 FORMAT(2F10.0)
 2000 FORMAT('1***ERROR ICOUNT=10***',3E20.5)
 2010 FORMAT('0',15X,3E15.5)
 2020 FORMAT('1')
 2030 FORMAT(' ',E15.5)
      END
```

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G5414 Golden
A numerical method for two-dimensional, cavitating, lifting flows.